"Computer-Aided Analysis of Electronic Circuits"
Course Notes 2

Bachelor: Telecommunication Technologies and Systems
Year of Study: 2

Lecturer: Dan Burdia, PhD
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Chapter II - Computer Circuit Models of Electronic Devices and Components.

- 2.1 Basic set of devices for circuit modeling
- 2.2 Hierarchies and types of models
- 1.3 Considerations on model development
- 1.4 Examples of device modeling (diode, bipolar transistor)
1.1 Basic set of devices for circuit modeling

Basic set includes 5 types of devices

- **R** (resistor)
  - Linear resistor: \( v = R i \), \( i = G v \)
  - Non-linear resistor: ex: \( i = g(v) \)
- **L** (inductor) – linear or non-linear
- **C** (capacitor) – linear or non-linear
- Independent sources
  - Voltage source
  - Current source
- Controlled sources
  - Voltage controlled
    - \( E \) – voltage source; \( G \) – current source
  - Current controlled
    - \( H \) – voltage source; \( F \) – current source
2.2 Hierarchies and types of models

- **Criterias: dynamic range, frequency bandwidth**
- **Dynamic range criteria**
  - Global models
  - Local model
  - Linear-incremental models
- **Frequency bandwidth criteria**
  - DC models
  - AC models
2.2.1 Dynamic range criteria - example of models for BT

Bipolar transistor
Schematic symbol

Exact static output characteristics

Global model

Local model

Linear-incremental model (small signal model)
2.2.2 Frequency range criteria - example

Thin film chip resistor models

- DC Model
- Low Frequency Model
- High Frequency Model
- High Frequency Model with external inductance and capacitance

Symbols:
- R: resistance
- L: internal inductance
- C: internal shunt capacitance
- L_{C}: external connection inductance
- C_{G}: external capacitance to ground
2.2.3 Model hierarchy

AC Global Model → AC Local Model → AC Linear Model

DC Global Model → DC Local Model → DC Linear Model

AC Global Model = Large Signal Model
AC Linear Model = Small Signal Model
2.3 Foundation of model development

- No matter what approach is used to construct a model for a physical device, its validity depends on how well the model does indeed predict the behavior of the physical device.

- **Validity of the models for linear systems**

- **Representation Theorem for Linear Systems:** Let $Na$ and $Nb$ be two $(n+1)$-terminal linear black boxes. Suppose that both $Na$ and $Nb$ are driven by $n$ arbitrarily prescribed **sinusoidal** sources and suppose that the sources connected to corresponding terminals of $Na$ and $Nb$ are identical. If the complete response of the corresponding terminals of $Na$ and $Nb$ are identical under the described excitation scheme, then $Na$ and $Nb$ are equivalent to each other in the sense that they will always possess identical response under any other corresponding excitations.
2.3 Foundation of model development

- **Validity of the models for non-linear systems**
  - Can be established the validity of the DC Global and DC Local Models

  - *For non-linear AC Global and AC local models the characteristic curves depend on frequency, they are not unique.*

- Two approaches are used for synthesizing AC non-linear models
  - Physical approach
  - Black-box approach

- **Physical approach:** starting from physical structure and operating mechanisms of a given device to obtain a circuit model.

- **Black-box approach:**
  - Firstly, a valid DC global model is obtained.
  - Then, capacitors and inductors are added at one or more strategic locations to assure the behavior in the frequency domain.
2.3 Foundation of model development

Example: Thermistor circuit model

- Physical approach: Resistance – Temperature variation
  \[ v = R_0(T_0) \exp \left[ \beta \left( \frac{1}{T} - \frac{1}{T_0} \right) \right] \cdot i \triangleq R(T) \cdot i \]

- Black-box approach: Power dissipation – Temperature variation
  \[ p(t) = v(t) \cdot i(t) = \delta(T - T_0) + C \frac{dT}{dt} \]

- Circuit model
2.4 Examples of semiconductor device modeling

- 2.4.1 Diode circuit model
- 2.4.2 Bipolar transistor circuit model

**Diode circuit model**

- AC global model

![Schematic symbol](image) \[\implies\] \[\text{Diode AC global model}\]

- RS - series resistor of diode terminals
- \( I_D(\text{V}_D) \) - Voltage-Controlled Current Source describing I-V diode DC characteristic
- \( C_j \) - Barrier capacitance
- \( C_d \) - Diffusion capacitance
Diode circuit model

DC Global model
Diode circuit model

DC Global model - I-V expressions

\[ I_D = A \cdot (I_{fwd} - I_{rev}) \]

Forward current

\[ I_{fwd} = I_{nrm} K_{inj} + I_{rec} K_{gen} \]

Normal current

\[ I_{nrm} = I_S \cdot \left[ \exp\left( \frac{V_D}{N \cdot U_T} \right) - 1 \right] \]

Recombination current

\[ I_{rec} = I_{SR} \left[ \exp\left( \frac{V_D}{NR \cdot U_T} \right) - 1 \right] \]

\[ K_{inj} = \begin{cases} \sqrt{\frac{I_{KF}}{I_{KF} + I_{nrm}}} & \text{dacă } I_{KF} \geq 0 \\ \frac{1}{I_{KF} + I_{nrm}} & \text{dacă } I_{KF} < 0 \end{cases} \]
Diode circuit model

DC Global model - I-V expressions

Reverse current

\[ I_{\text{rev}} = I_{\text{rev\_high}} + I_{\text{rev\_low}} \]

\[ I_{\text{rev\_high}} = IBV \cdot \exp \left( -\frac{V_D + BV}{NBV \cdot U_T} \right) \]

\[ I_{\text{rev\_low}} = IBVL \cdot \exp \left( -\frac{V_D - BV}{NBVL \cdot U_T} \right) \]

BV – breakdown voltage

IBV, IBVL, NBV and NBVL = diode model parameters
Diode circuit model

Capacitance modeling

\[ C = C_d + A \cdot C_j \]

\( C_d \) – diffusion capacitance

\[ C_d = \tau_D \frac{dI_{fwd}}{dV_D} \]

\[ \tau_D = \frac{1}{2\pi F} \quad (\text{transition time}) \]

\( C_j \) – barrier capacitance

\[ C_j = \frac{C_{JO}}{\left(1 - \frac{V_D}{V_J}\right)^M} \quad (\text{available for negative or small values of } V_D) \]
Diode circuit model

SPICE model of Barrier Capacitance

\[
C_j = \begin{cases} 
C_{JO} \cdot \left(1 - \frac{V_D}{V_J}\right)^{-M}, & V_D < FC \cdot V_J \\
\frac{C_{JO}}{(1 - FC)^{M+1}} \cdot \left(\frac{M \cdot V_D}{V_J} + 1 - FC(1 + M)\right), & V_D \geq FC \cdot V_J 
\end{cases}
\]
Diode circuit model

Small signal model

\[ g_D = \left. \frac{dI_D}{dV_D} \right|_{PSF} \]

\[ C_d = \tau_D \cdot \left. \frac{dI_D}{dV_D} \right|_{PSF} = \tau_D \cdot g_D \]

Noise modeling

\[ \overline{i_{ZS}^2} = \frac{4kT}{(R_S / Aria)} \cdot \Delta f \]

\[ \overline{i_{ZD}^2} = 2qI_D \Delta f + KF \cdot \frac{I_D^{AF}}{f} \Delta f \]
Diode circuit model

Temperature variation

\[
R_s(T) = R_s \cdot \left[1 + \text{TRS}1 \cdot (T - T_{\text{nom}}) + \text{TRS}2 \cdot (T - T_{\text{nom}})^2 \right]
\]

\[
I_s(T) = I_s \cdot \exp \left[ \left( \frac{T}{T_{\text{nom}}} - 1 \right) \frac{E_G}{N \cdot U_T} \right] \left( \frac{T}{T_{\text{nom}}} \right)^{X_{TI}}
\]

\[
BV(T) = BV \cdot \left[1 + \text{TBV}1 \cdot (T - T_{\text{nom}}) + \text{TBV}2 \cdot (T - T_{\text{nom}})^2 \right]
\]

\[
C_{JO}(T) = C_{JO} \cdot \left\{1 + M \cdot \left[4 \cdot 10^{-4} \cdot (T - T_{\text{nom}}) + \left(1 - \frac{V_j(T)}{V_j}\right)\right]\right\}
\]
SPICE model of Bipolar transistor

Bipolar transistor circuit model

DC current expressions

\[ I_b = A \cdot \left( I_{be1} / BF + I_{be2} + I_{bc1} / BR + I_{bc2} \right) \]

\[ I_c = A \cdot \left( I_{be1} / K_{qb} - I_{bc1} / K_{qb} - I_{bc1} / BR - I_{bc2} \right) \]

\[ I_{be1} = I_s \cdot \left[ \exp \left( \frac{V_{be}}{NF \cdot U_T} \right) - 1 \right] \]

\[ I_{be2} = I_{se} \cdot \left[ \exp \left( \frac{V_{be}}{NE \cdot U_T} \right) - 1 \right] \]

\[ I_{bc1} = I_s \cdot \left[ \exp \left( \frac{V_{bc}}{NR \cdot U_T} \right) - 1 \right] \]

\[ I_{bc2} = I_{sc} \cdot \left[ \exp \left( \frac{V_{bc}}{NC \cdot U_T} \right) - 1 \right] \]
SPICE model of Bipolar transistor

Capacitances modeling

Base-emitter capacitance

\[ C_{be} = C_{dbe} + A \cdot C_{jbe} \]

- Base-emitter diffusion capacitance

\[ C_{dbe} = t_f \cdot \frac{dI_{be}}{dV_{be}} \]

- Base-emitter barrier capacitance

\[ C_{jbe} = \begin{cases} 
C_{JE} \cdot \left( \frac{V_{be}}{V_{JE}} \right)^{-MJE} , & V_{be} < FC \cdot V_{JE} \\
\frac{C_{JE}}{(1 - FC)^{MJE+1}} \cdot \left( \frac{MJE \cdot V_{be}}{V_{JE}} + 1 - FC(1 + MJE) \right) , & V_{be} \geq FC \cdot V_{JE}
\end{cases} \]

Collector-emitter capacitance - \( C_{bc} \) has similar expressions as \( C_{be} \)