Fuzzy evaluation processing in Decision Support Systems.

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Abstract. We propose an algorithm of forming orders portfolio when the funds are limited and multicriterion evaluations of projects by experts are fuzzy. The algorithm is used in DSS for planning of financing of scientific researches.
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1. Introduction

The concept of decision support system (DSS) permits to combine harmoniously strict mathematical methods of optimization with the intuition and reasoning ability of the decision maker. In most cases, when solving practical problems, the manager has to deal with incomplete and, especially, inexact data and to make decision on their base. That is why it seems to be not very natural to use, when solving problems of optimization or simulation, common methods elaborated for exact data.

The worked out by us DSS for forming portfolio of projects awaiting, financing includes also an algorithm based on the approach which uses fuzzy sets.

2. Problem statement

Every year scientific teams and individual scientists submit their project to different foundations and state institutions for the purpose of obtaining finances. As a rule, the projects are sent to experts who evaluate them using criteria determined in advance.

Let N projects be admitted to the competition. On the finite set $I = \{i_1, i_2, \ldots, i_N\}$ of the projects *m* criteria (quantitative or qualitative) $c_1, c_2, \ldots, c_m$, each of them taking values from an ordered set. The experts evaluation can give as result some aggregated indices that show the effect e_i^t from the realization of the *i*-th project in case the project will be given finances at the moment under consideration. It is natural that in the orders portfolio those projects should be included which will provide the maximal efficiency and whose realization does not require the exceeding of envisaged funds.

3. Algorithms

The projects assume different terms of realization. It is supposed that the financing plan is worked out annually, allocating the volume M of finances for the first year. The term of the realization of the *i*-th project is T_i. Each stage of the realization of the *i*-th projects requires the sum of $m_i^t (t = 1,2,\ldots,T_i)$. The total sum for the realization of the *i*-th proiect

$M_i = \sum_{i=l} m_i^t$. The selection of projects to be included in the financing plan envisages two stages:

– analysis and evaluation of projects by experts,
– decision making.

At the first stage the set I of projects is divided into subsets $I_k = \{i_1, i_2, \ldots, i\bar{k}\}$, $k = 1,2\ldots\bar{k}$, consisting of rival projects, i.e. those ones that solve the same problem.

Let $e_{ik}^t (i \in I_k, k = 1,2,\ldots\bar{k},\ t = 1,2,\ldots,T_i)$ be the effect from the realization of the i-th project of the subset I_k after t years if the project will be financed. These numbers we regard as fuzzy and they are determined by experts at the first stage of the evaluation of given information.

Let A and B two fuzzy numbers with membership functions μ_A and μ_B respectively, then the membership functions of the numbers C = max (A,B) and D = A+B are determined as follows:

$$\mu_{C(z)} = \max_{z=max(x,y)} \min \{\mu_A(x), \mu_B(y)\}$$

and

$$\mu_{D(z)} = \max_{z=x+y} \min \{\mu_A(x), \mu_B(y)\}$$

.

Let x_i be the variable which is equal to 1 if the project I is included in the financing plan and is equal to 0 otherwise. Then the efficiency of the realization of the projects from the subset I_k that get financing after t years can be calculated as follows:

$$E_k^i = \max_{i \in I_k} e_{ik}^t \cdot x_i,\ \ t = 1,2\ldots t_k \ \text{ where } \ t_k = \max_{i \in I_k} T_i .$$

I_f $t_k \geq T_i$ for some $i \in I$, we will consider that $e_{ik}^t = e_{ik}^{T_i}$, for all $t \geq T_i$.

Then the total effect of the variant of the plan aftert years will be

$$E^t = \sum_{k=1} E_k^t \quad t = 1, \ 2,\ldots, t_k .$$

Taking in consideration the limited funds for the first year, we can formulate the problem [1]:

$$\begin{cases} E^t \longrightarrow \max, \\ \sum_{i \in I} m_i^1 \\ x \in \{0,1\} \end{cases} \qquad (1)$$

The following algorithm for solving problem (1) is proposed.

The decision make desired levels for each $E^t, t = 1,2,\ldots,t$.

We obtain the following system of inequahities:

$$\begin{cases} E^t \geq \tau_t, t = 1,2,..,\tau \\ \sum_{i \in I} m_i^t \leq M \\ x_i \in \{0,1\}, i = 1,2,...,N \end{cases} \quad (2)$$

The solutions of systems (2) may be consdered ad missible solutions of problem (1). As the inequalities of system (2) are fuzzy, its solutions are fuzzy sets, too. It is obvions that it is desirable to choose the solution thet maximizes the membership function.

Let, for $t = 1,2,...,t$, $A^t = \{y : E^t \geq y\}$. It is clear that A^t is a fuzzy set with the membership function.

$$\mu_A{}^t(y) = \max_{z \geq y} \mu_E{}^t(z).$$

Remark that $\mu_A{}^t(\iota_t) = \mu_D{}^t(x)$, where D^t is the set of solutions of problem (1). Therefore, we obtain the following problem:

$$\begin{cases} \min_i \max_{z \leq z_1} \mu_E{}^t(z) \to max \\ \sum_{i \in I} m_i^t M \\ x_i \in \{0,1\}, i = 1,2,...,N \end{cases} \quad (3)$$

The solution of problem (3) is the vector $x^* = (x_1^*, x_2^*,..., x_N^*)$.

The value of function $l^* = l(x^*)$ may be interpreted as the degree of the membership of x^* to the set of solutions of problem (1).

The decision maker, varying the levels r_t, can obtain the financing plan determined by the vector x^* that satisfies it.

4. Conclusions.

We don't consider here a vory important stage of the expert analysis when the evaluations e_{ik}^t are determined. We would like to mention only that every such evaluation e_{ik}^t is the result of a set of fuzzy evaluations corresponding to a group of experts.

References

[1] *Gaindric C., Spac A., Zaporojan D. A decision system in complex problems. The N-th International Symposium on Fuzzy Systems and Artificial Intelligence, Iasi (Romania), 1991, pp. 121-125.*