Systematic Approach to Nonlinear Modelling Using Fuzzy Techniques

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Abstract. The paper deals with the systematic approach of using Fuzzy Models as universal approximators. Four types of models suitable for identification are presented: The Nonlinear Output Error, The Nonlinear Input Error, The Nonlinear Generalized Output Error and The Nonlinear Generalized Input Error Model. The convergence properties of all four models in the presence of disturbing noise are reviewed and it is shown that the condition for an unbiased identification is that the disturbing noise is white and that it enters the nonlinear model in specific point depending on the type of the model. The application of the proposed modelling approach is illustrated with a fuzzy model based control of a laboratory scale heat exchanger.

1. Introduction

Fuzzy logic provides a mathematical tool to formulate mental models in a compact mathematical form. The intuitive and heuristic nature of the human mind, which is actually imprecise, can be incorporated in formal models which can essentially support the planning and decision making processes. Fuzzy logic has extended the classical mathematical models in the form of differential and difference equations to a broad class of models which are easily understandable.

An overview and classification of identification models can be found in [1]. Fuzzy logic originates from so called "intelligent" control and can be treated as an universal approximator (denoted by UA in the sequel) which can approximate continuous functions to an arbitrary precision [2], [3] [4]. In general UA may have several inputs and outputs. Without loss of generality, only one output will be treated; the approximators with more than one output can be treated as several approximators in parallel.

The paper is organized as follows: Different types of fuzzy models will be presented as UA and their application to dynamic models will be introduced in the next Section. In Section 3 four basic forms suitable for the identification of nonlinear dynamic models will be given and their convergence properties regarding the disturbing noise will be examined in Section 4. Finaly the application of the proposed modelling approach will be illustrated in the Section 5 as a case study of the fuzzy model based control application to a laboratory scale heat exchanger.

2. Rule Based Nonlinear Dynamic Models

According to [5], three types of "rule based" fuzzy models are known: the linguistic, the fuzzy relational and the Takagi - Sugeno fuzzy models [6]. A typical Takagi-Sugeno type rule can be written

$$R^j : \textbf{if } x_1 \textbf{ is } A_1^j \textbf{ and} \ldots \textbf{and } x_N \; is \; A_N^j \quad \textbf{then} \quad y = g^j(x_1 \ldots x_N) \tag{1}$$

where $x_1 \ldots x_N$ are inputs, A_1^j a subset of the input space, y output and g^j a function (in general nonlinear, usually linear). In this case the output of the UA with n inputs can be written in the following form

$$y(k) = \frac{\sum_{i1}, \sum_{i2}, \ldots, \sum_{in} s_{i1,i2,\ldots,in}(\mathbf{x}) r_{i1,i2,\ldots,in}}{\sum_{i1}, \sum_{i2}, \ldots, \sum_{in} s_{i1,i2,\ldots,in}} \tag{2}$$

where $s_{i1,i2,\ldots,in}$ is the element of the multidimensional structure (tensor)

$$S = \mu_1 \otimes \mu_2 \otimes \ldots \otimes \mu_n \tag{3}$$

which is obtained by the composition $\otimes$ (usually a product or an other T norm) of fulfilment grade vectors (of dimensions mi) of membership functions on the universe of discourse

$$\mu_i = [\mu_i^1, \mu_i^2 \ldots \mu_i^{mi}]^T \quad . \tag{4}$$

where mi is the number of membership functions. $r_{i1,i2,\ldots in}$ are consequences of the Takagi-Sugeno type model according to Eq. (1). In the simplest case they are constants (Takagi - Sugeno models with crisp constant consequences).

Piece-wise linear interpolation between the rule consequents is obtained if the following conditions hold [7]

1. The antecedent membership functions are triangular and form a partition, i. e. $\sum_{j=1}^{mi} \mu_i^j = 1, \forall i$.
2. The product T norm is used to represent the logical "and", and the rule base is complete (i.e. rules are defined for all combinations of the antecedent terms).
3. The fuzzy-mean defuzzyfication is applied.

Under condition 1. the denominator of Eq. (2) becomes 1.

However even with the Mamdani type rules where the consequence is symbolic (fuzzy set):

$$R^j : \quad \textbf{if } x_1 \textbf{ is } A_1^j \textbf{ and} \ldots \textbf{and } x_N \textbf{ is } A_N^j \quad \textbf{then} \quad y \; is \; B^j \tag{5}$$

the fuzzy system (fuzzyfication - inference - defuzzyfication) can be treated as nonlinear mapping between inputs and outputs. So in general we may write

$$y(k) = f\Big(\mathbf{x}(k)\Big) \quad . \tag{6}$$

The input-output relations of industrial processes and controllers are dynamic. A common approach to dynamic models is to use time shifted signals, which result in discrete-time models. The usage of derivatives, integrals or other transfer function would result in continuous-time models. Also, mixed-continuous-discrete time models are possible. In this paper the discrete time models will be treated and tapped-lines will be used to generate the time-shifted signals. If only the (tapped) input signal is used as input of the UA (elements of $\mathbf{x}$), the resulting model is nonrecursive and has finite impulse response. Such models are a class of Nonlinear Finite Impulse Response models and are denoted by NFIR. If the (tapped) input and (tapped) output signals are used as input of the UA, the resulting model is recursive and may have an infinite impulse response. Four types of such models will be given in the next Section.

3. Systematic Approach to the Identification Models

Experimental modelling or identification is based on observation of input-output data. In the sequel it will be supposed that the unknown plant which has produced the input-output data can be described in the following mathematical form

$$\begin{aligned} y(k) &= F\Big(u(k),\ldots,u(k-N_p),y(k-1),\ldots\ldots,y(k-N_p),v(k)\Big) \\ &= f\Big(u(k),\ldots,u(k-N_p),y(k-1),\ldots\ldots,y(k-N_p)\Big)+n(k) \end{aligned} \tag{7}$$

where $F()$ and $f()$ are unknown nonlinear functions, N_p the order of the model, $v(k)$ the noise disturbing the nonlinear plant and $n(k)$ the nonlinear noise at the plant output. The resulting model is obtained by the best fit of the model response to the identified process response if the same input signal is applied to both of them. So the identification problem is formulated as an optimization problem utilising a criterion in the form of a functional e.g. the sum of squared errors

$$E(y,y_M)=\sum_{k=0}^{N}\epsilon^2(k) \tag{8}$$

where y is the observed signal, y_M is the model output and ϵ the error. The identification procedure involves the structure identification of the plant and the estimation of unknown parameters. In practice, the structure is usually chosen ad hoc (engineering feeling) and then improved by an optimization procedure. Optimization is also used for the determination of parameters. Four cases of model description will be given next.

1. The case

   $$\epsilon(k)=y(k)-y_M(k) \tag{9}$$

   where

   $$y_M(k)=\hat{f}\Big(u(k),\ldots,u(k-N),y_M(k-1),\ldots,y_M(k-N)\Big) \tag{10}$$

   is the output of a recursive model with input $u(k)$, is referred to as **Nonlinear Output Error Model**. The nonlinear function $\hat{f}()$ is an estimate of the nonlinear function $f()$ and in ideal case, where the plant is identified perfectly, both nonlinear functions become the same ($\hat{f}()=f()$).

2. The case

$$\epsilon(k) = u(k) - u_M(k) \tag{11}$$

where

$$u_M(k) = \hat{f}_i\Big(y(k), \ldots, y(k-N), u_M(k-1), \ldots, u_M(k-N)\Big) \tag{12}$$

is the output of a recursive model with input $y(k)$, is known as **Nonlinear Input Error Model**. The nonlinear function $\hat{f}_i()$ represents the estimation of the nonlinear function $f_i()$ which is the inverse of the function $f()$ in the sense that if the signal $y(k)$ is produced by Eq. (7) and no noise is present ($n(k) = 0$), then the signal produced by the inverse function is identical to the signal $u(k)$

$$u(k) = f_i\Big(y(k), \ldots, y(k-N_p), u(k-1), \ldots, u(k-N_p)\Big) \tag{13}$$

The inverse function f_i exists if the function $f()$ is injective. Also only the dynamic part of the plant without time delay is inverted. In ideal case where the plant is identified perfectly both inverse nonlinear functions become the same ($\hat{f}_i() = f_i()$).

3. If in the right hand side of Eq. (10) the output of the model $y_M(k)$ is replaced by the measured output of the process, i.e.

$$y_M(k) = \hat{f}\Big(u(k), \ldots, u(k-N), y(k-1), \ldots, y(k-N)\Big) \tag{14}$$

the error defined by Eq. (9) becomes a **Nonlinear Generalized Output Error Model**. It has two inputs, namely the plant input signal $u(k)$ and the plant output signal $y(k)$ and one output signal $y_M(k)$. The nonlinear function $\hat{f}()$ is an estimate of the nonlinear function $f()$ and in ideal case where the plant is identified perfectly both nonlinear functions become the same ($\hat{f}() = f()$).

4. If in the right hand side of Eq. (12) $u_M(k)$ is replaced by the input of the process, i.e.

$$u_M(k) = \hat{f}_i\Big(y(k), \ldots, y(k-N), u(k-1), \ldots, u(k-N)\Big) \tag{15}$$

the error defined by Eq. (11) becomes a **Nonlinear Generalized Input Error Model**. It has two inputs, namely the plant output signal $y(k)$ and the plant input signal $u(k)$ and one output signal $u_M(k)$. The nonlinear function $\hat{f}_i()$ represents the estimation of the nonlinear function $f_i()$ and in ideal case where the plant is identified perfectly both inverse nonlinear functions become the same ($\hat{f}_i() = f_i()$).

Fig. 1 represents all four cases, the Nonlinear Output, Input and both Generalized Error Identification Models respectively.

All four forms of the error models are closely related. The input error and the Generalized input error models are inverse to the output error and the Generalized

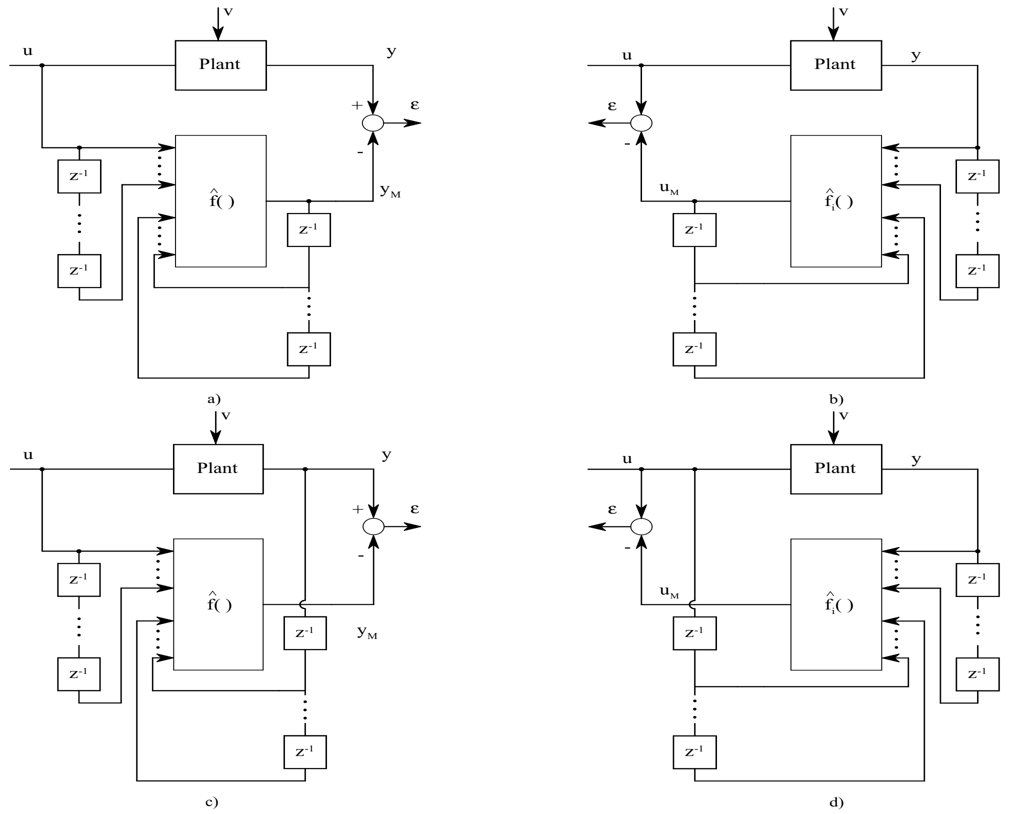

Figure 1: The output error (a), the input error (b), the generalized output error (c) and the generalized input error (d) identification models.

output error models respectively in the sense described above. The output and input models are complementary to the Generalized output and Generalized input models respectively in the sense that Generalized models are suitable for identification since with known or supposed structure of the nonlinearity (which remain fixed during the optimization) the estimation of unknown parameters becomes a linear problem and the least squares technique can be used. This is the case of Takagi-Sugeno type models with Center of Singletons defuzzification and predetermined fuzzy sets of the premise space (input partition) or the case of the radial base function networks with preselected locations of the regressors. The output/input models are applicable in prediction. So they are called also prediction or simulation models.

The four models described by Eqns. (10, 12, 14, 15) can be treated as universal dynamic approximators (UDA) where the nonlinear function is realised by fuzzy logic universal approximators (UA). According to the topology of the models in Fig. 1 the input, output and both Generalized models can be called also parallel, series and series-parallel models respectively. It should be noted that there are two kinds of the Generalized error (series-parallel) models, origination in the output and input error models respectively.

4. Convergence properties

An important point of identification is the convergence in the presence of the noise. Since the white noise has minimal variance all optimization procedures seek the minimum of the criterion function (8) in the sense that the residuals $\epsilon(k)$ become white noise. So an unbiased estimation is possible only in cases where the noise has a special character which will be discussed next for all four models

1. **Nonlinear output error model**. In this case the noise $v(k)$ is introduced additively to the output of the undisturbed plant $y_o(k)$

$$\begin{aligned} y_o(k) &= f\Big(u(k), \dots, u(k-N_p), y_o(k-1), \dots, y_o(k-N_p)\Big) \\ y(k) &= y_o(k) + v(k) \end{aligned} \tag{16}$$

   The residual $\epsilon(k)$ becomes now

$$\begin{aligned} \epsilon(k) =& y(k) - y_M(k) = \\ =& f\Big(u(k), \dots, u(k-N_p), y_o(k-1), \dots, y_o(k-N_p)\Big) + v(k) - \\ &- \hat{f}\Big(u(k), \dots, u(k-N), y_M(k-1), \dots, y_M(k-N)\Big). \end{aligned} \tag{17}$$

   If the structure of the identification model is the same as the structure of the nonlinear plant, the optimization procedure for the minimization of (8) tries to make the residual $\epsilon(k)$ white, so if $v(k)$ is white noise, the minimum is $\epsilon(k) = v(k)$ and consequently $\hat{f}() = f()$ and $y_M(k) = y_o(k)$. The condition for an unbiased estimation is that the noise is additively added to the plant output and that it is white as illustrated in Fig. 2-a.

2. **Nonlinear input error model**. In this case the noise $v(k)$ is introduced additively to the input of the undisturbed plant $u_o(k)$

$$\begin{aligned} u_o(k) &= f_i\big(y(k), \dots, y(k-N_p), u_o(k-1), \dots, u_o(k-N_p)\big) \\ u(k) &= u_o(k) + v(k) \end{aligned} \tag{18}$$

   The residual $\epsilon(k)$ becomes now

$$\begin{aligned} \epsilon(k) =& u(k) - u_M(k) = \\ =& f_i\big(y(k), \dots, y(k-N_p), u_o(k-1), \dots, u_o(k-N_p)\big) + v(k) - \\ &- \hat{f}_i\big(y(k), \dots, y(k-N), u_M(k-1), \dots, u_M(k-N)\big) \quad . \end{aligned} \tag{19}$$

   If the structure of the identification model is the same as the structure of the nonlinear plant, the optimization procedure for the minimization of (8) tries to make the residual $\epsilon(k)$ white, so if $v(k)$ is white noise, the minimum is $\epsilon(k) = v(k)$ and consequently $\hat{f}_i() = f_i()$ and $u_M(k) = u_o(k)$. The condition for an unbiased estimation is that the noise is additively added to the plant input and that it is white as illustrated in Fig. 2-b.

3. **Nonlinear Generalized output error model**

   In this case the noise $v(k)$ is introduced inside the plant as illustrated in Fig. 2-c. The plant equation becomes now

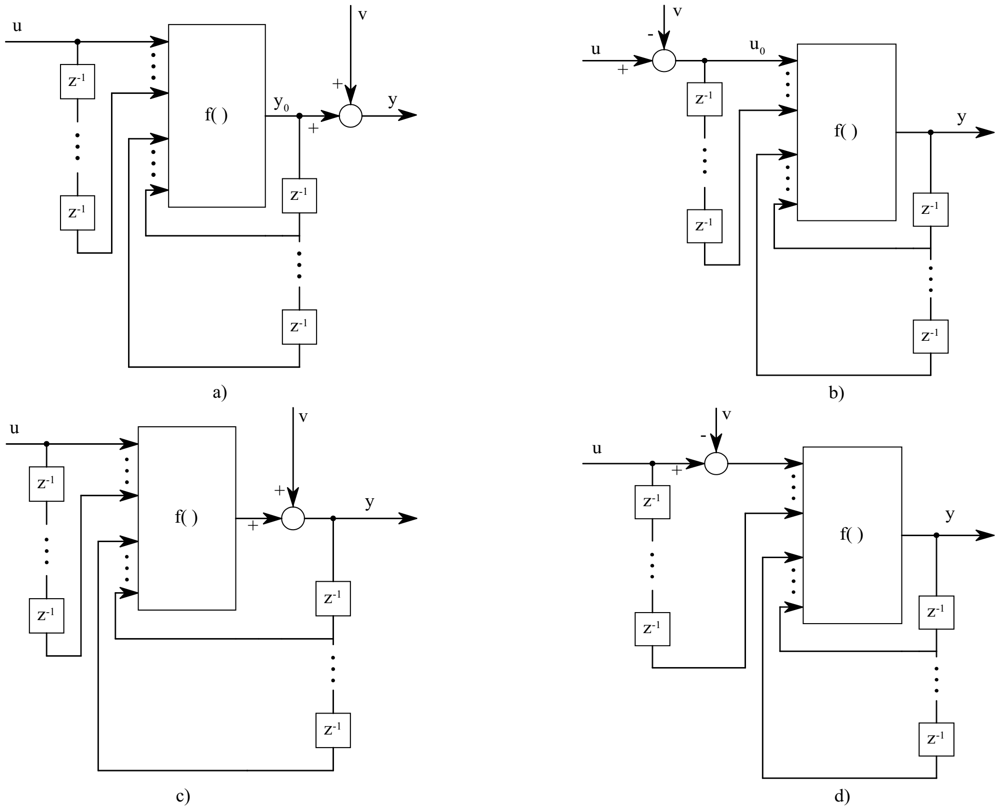

Figure 2: The representation of noise which results in an unbiased estimation for the output error (a), the input error (b), the generalized output error (c) and the generalized input error (d) models.

$$y(k) = \hat{f}\Big(u(k), \dots, u(k-N_p), y(k-1), \dots, y(k-N_p)\Big) + v(k) \tag{20}$$

and the residual

$$\begin{aligned}\epsilon(k) =& y(k) - y_M(k) = \\ =& f\Big(u(k), \dots, u(k-N_p), y(k-1), \dots, y(k-N_p)\Big) + v(k) - \\ & - \hat{f}\Big(u(k), \dots, u(k-N), y(k-1), \dots, y(k-N)\Big) \quad .\end{aligned} \tag{21}$$

If the structure of the identification model is the same as the structure of the nonlinear plant, the optimization procedure for the minimization of (8) tries to make the residual $\epsilon(k)$ white, so if $v(k)$ is white noise, the minimum is $\epsilon(k) = v(k)$ and consequently $\hat{f}() = f()$. The condition for an unbiased estimation is that the noise additively added to the plant as illustrated in Fig. 2-c is white.

4. **Nonlinear Generalized input error model** In this case the noise $v(k)$ is introduced inside the plant as illustrated in Fig. 2-d. The plant equation becomes now

$$u(k) = f_i\big(y(k), \dots, y(k-N_p), u(k-1), \dots, u(k-N_p)\big) + v(k) \tag{22}$$

and the residual

$$
\begin{aligned}
\epsilon(k) =&u(k) - u_M(k) = \\
&=f_i\big(y(k), \dots, y(k-N), u(k-1), \dots, u(k-N)\big) + v(k)- \\
&\quad - \hat{f}_i\big(y(k), \dots, y(k-N), u(k-1), \dots, u(k-N)\big) \quad .
\end{aligned} \tag{23}
$$

If the structure of the identification model is the same as the structure of the nonlinear plant, the optimization procedure for the minimization of (8) tries to make the residual $\epsilon(k)$ white, so if $v(k)$ is white noise, the minimum is $\epsilon(k) = v(k)$ and consequently $\hat{f}_i() = f_i()$. The condition for an unbiased estimation is that the noise additively added to the plant as illustrated in Fig. 2-d is white.

The estimation is unbiased if the structure of the model is the same as the structure of the plant and the white noise enters into the plant in a special location depending on the type of a model used in identification. With the output error model this is the output of the UDA; the plant output is in this case the sum of uncorrupted signal of UDA and the white noise. With the input error model this is the input of the plant; the input signal of the UDA is the sum of the uncorrupted input signal and the white noise. With Generalized output error model the white noise enters into the plant at the output of the UA; the corrupted output signal is fed back and represents the input of the UA. With Generalized input error model the white noise enters into the plant only at the first input of the UA.

5. Application to a Laboratory Scale Heat Exchanger

The identification models were applied to a fuzzy model based control of a laboratory scale heat exchanger. The accessory used, depicted schematically in Fig. 3, consists of a plate heat exchanger, through which hot water from an electrically heated reservoir is continuously circulated in counter-current flow to the cold process fluid (cold water). Thermocouples are located in the inlet and outlet streams of the exchanger, thus the primary and secondary flow rates can be visually monitored. Power to the heater is controlled by an external control loop. The flow of the heating fluid can be controlled by a proportional motor driven valve. The control variable is the control current of the valve (4-20 mA), while the controlled variable is the temperature of the water in the secondary circuit at the heat exchanger outlet.

The plant was identified off-line on the basis of signals which assure a throughout excitation. The sampling time used was 4 s. As the dynamics of the plant exhibits approximately first order dynamics with a small time delay, the three step delayed control variable and one step delayed controlled variable were used as inputs to the Nonlinear Generalized Output Error model

$$
y_M(k) = f\Big(u(k-3), y(k-1)\Big) \tag{24}
$$

If, on the right hand side of Eq. (24), $u(k-1)$ were used, the resulting nonlinear dynamic system would be of the first order without any delay. By delaying the control variable by two additional steps, the dead time of 8 s was achieved. Two Generalized Output Error models were compared. With the first one, the universe of discourse of the control and the controlled variables were divided into five triangular, equally spaced

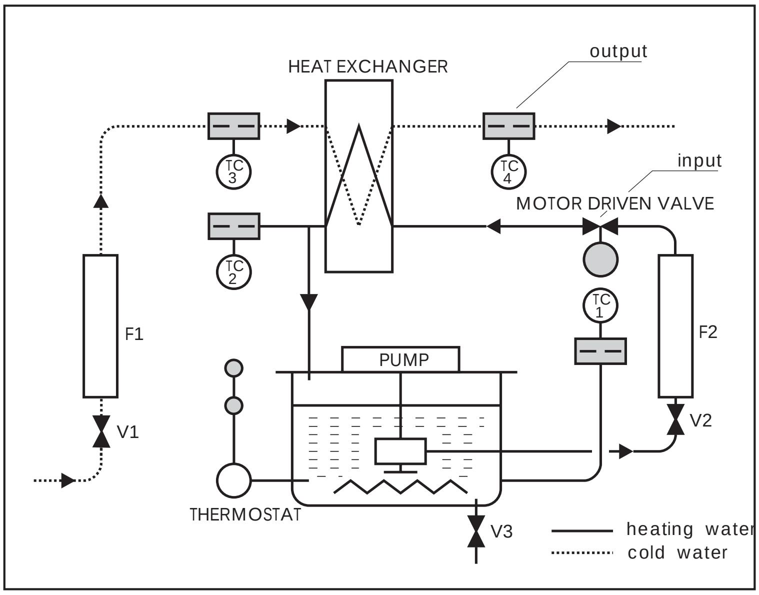

Figure 3: The scheme of the laboratory scale heat exchanger.

membership functions respectively using the Takagi-Sugeno model with crisp constant consequences ($f^j = c^j$). The central values of the membership functions for control and controlled variables were [4, 8, 12, 16, 20] mA and [13.00, 22.75, 32.50, 42.25, 52.00] ^{o}C, respectively. The product was used as the composition operator $\otimes$, and the rule base was complete.

In the second model, three triangular equally-spaced membership functions were used, together with the Takagi-Sugeno model with linear consequences ($f^j = c_1^j u(k-3) + c_2^j y(k-1) + c_3^j$). Both models have approximately the same number of parameters to be determined ($5 \times 5 = 25$ for the first, and $3 \times 3 \times 3 = 27$ for the second model, respectively). The Takagi-Sugeno model was chosen to fulfil the three conditions of Section 2.. The unknown parameters were estimated by the least squares method. In general, [9] first order consequences result in better tracking, however our result of the comparison was that the first model (five membership functions and crisp constant consequences) has a slightly better tracking capability. This somewhat surprising result was probably due to the high nonlinearity of the plant with which the three membership functions are hardly able to cope. For simplicity, the Takagi-Sugeno model with crisp constant consequences was used in the design and real time implementation of the control algorithm.

It should be noted that the Nonlinear Generalized Output Error model was used in the identification procedure, while the Nonlinear Output Error model was used for long range prediction, as required by the predictive control algorithm. This is a simple practical solution; theoretically, the Nonlinear Output Error model should be also used for the identification, however in this case the optimization problem would be nonlinear, with all the known effects.

The treated plant was first identified by a linear first order Generalized Output Error model with three steps time delay. The results of the validation are shown in Fig. 4

Next, the Nonlinear Generalized Output Error model was applied. Fig. 5 shows the validation of the applied model. It can be seen that the nonlinear model has much lower output error, which means much lower model uncertainty. Lower model uncertainty

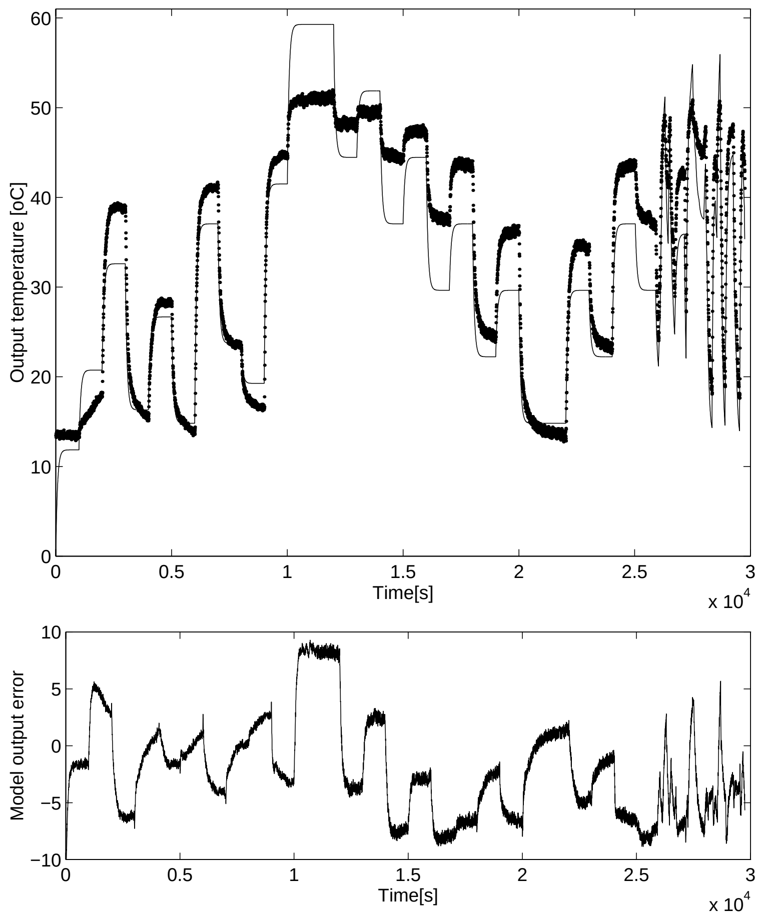

Figure 4: Comparison of the responses of the linear nominal model and the measured plant.

enables a choice of controller parameters which give better tracking quality.

The design parameters of the model based predictive control (the control N_1 and the prediction N_2 horizon, respectively, and the weight factor r) were $N_1 = 2$, $N_2 = 30$, $N_u = 3$ and $r = 0.002$

Fig. 6 depicts the closed loop response of the Nonlinear Output Error model-based predictive control. It can be seen that the closed loop response remains approximately the same throughout the entire range of the controlled variable, and that the response is stable and adequate.

6. Conclusion

Four forms of Fuzzy Identification Models are presented in the paper. Their convergence properties in the presence of noise are reviewed and it is shown that unbiased estimation is achieved if the disturbing noise is white and enters the plant at specific points: at the plant output for the Nonlinear Output Error Model, at the plant input for the Nonlinear Input Error Model, and inside the plant at the output and first input of the nonlinear block for Nonlinear Generalized Output Error and Nonlinear Generalized Input Error Models respectively. In the case of fixed (known or preselected) structure the estimation becomes linear in unknown parameters. An application of the proposed modelling technique to the laboratory scale heat exchanger illustrates the practicability of the proposed design technique.

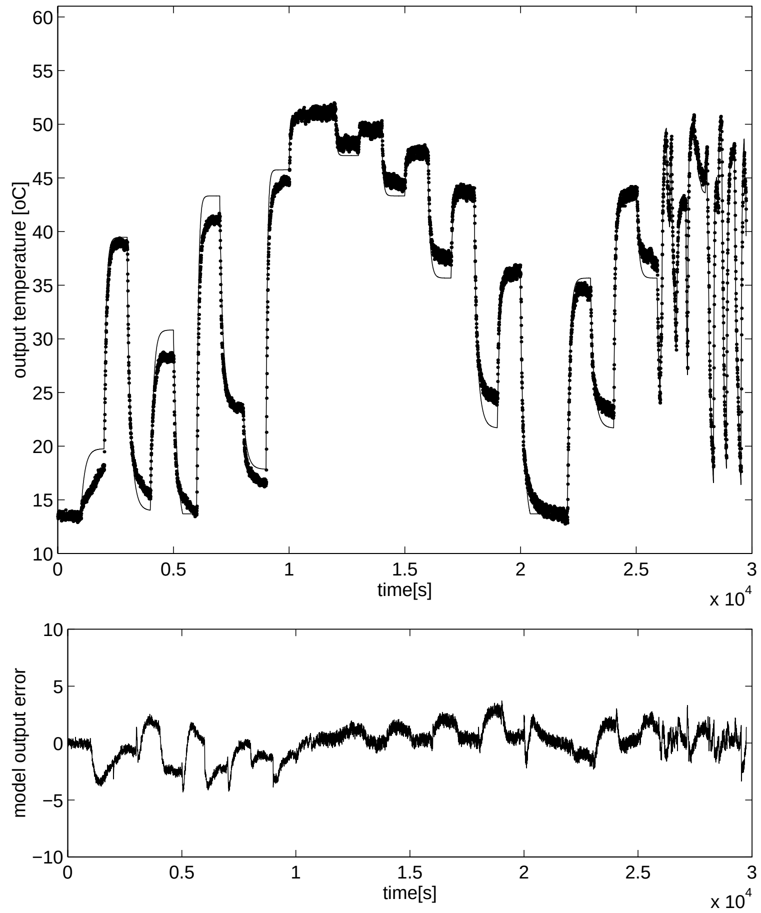

Figure 5: The validation of the Nonlinear Generalized Output Error model.

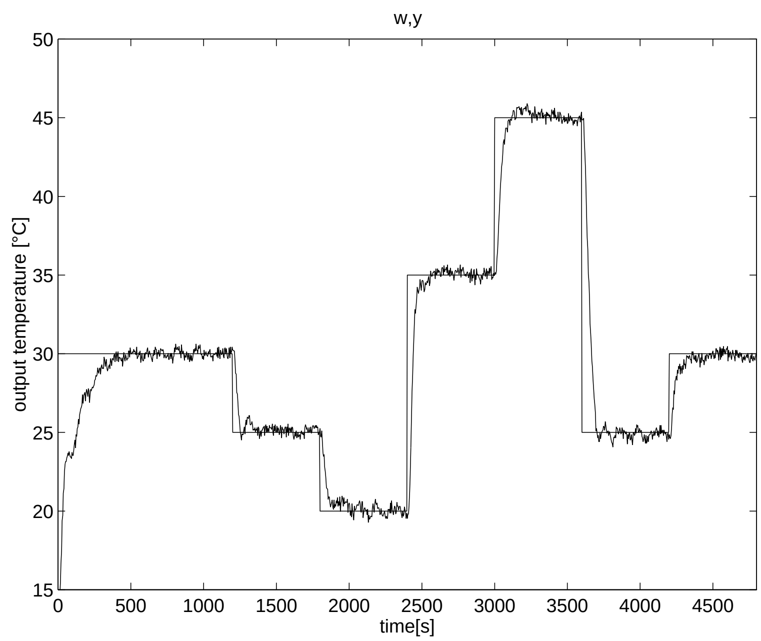

Figure 6: The closed loop response of the Nonlinear Output Error model based predictive control.

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