Uncertainty and Vagueness in Information

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" stochastics is the best tool available for handling likelihood ...

fuzzy set theory is another tool used to deal with uncertainty ..."

(J. Intel. & Fuzzy Systems)

One can make comparisons

between the trees in the forest

and wolves in the forest

but

but I would like to protest

the plan to this is as follows.

- 1. Simple example demonstrating distinct features of certainty, uncertainty and vagueness
- 2. Simple characterisation of two empirical phenomena uncertainty and vagueness
- 3. Simple example of an ideal mode for systematic presentation of information : axiomatic theory
- 4. A bird's eye view on stochastics

CERTAINTY I know I will take out a Polish coin from my pocket UNCERTAINTY Before casting I do not know what will result VAGUENESS After looking at it I do not know whether is it small

From a REISTI	C point of view a	fragment of reality	
observable by us consists of a large number of separate			
things with different properties, connected by			
different relations, making up different sets			World of WORDS
Information which can be proved or derived by means			
of valid logical arguments is called certain information.			
"We know something with certainty, and			
we conjecture about the things when we are not certain"			
rs cogitandi Ars conjectandi			
1662 : A. Arnauld, P.	Nicole	1713 : J. Bernoull	i
Adequatio rei et intellectus			
VERITAS	VERISIMILIS		
truth	probat	probability	
	epistemic	aleatory	
The results of observation, thinking and conjecturing			
have to be cast into linguistic form:			
particularly in the form of propositions			
regarded as conceptu	al reconstruction of	f certain	
traits of reality			
Logic	Stochastics		Fuzziness

Uncertainty

exists because of a lack of biunivocal correspondence between causes and consequences

there are limits for certainty

pertains the WORLD

refers to prediction, reasoning

this is my defect my ignorance

probability $F_x(x)$

probability is warranted by evidence: evidence by testimony evidence by things Vagueness

exists because of lack of sharp definitions

continuous semantic of natural languages

there are no limits for sharpening

pertains the WORDS

refers to classification, discrimination

this is "your" carelessness in naming my doublet (in applicability of the words)

applicability $\mu_X(x)$

applicability is warranted by convention

LOGIC

is the **systematic** study of

- the techniques for formulating **information** in language
- the methods of extracting information from linguistic formulation

L1.
$$\alpha \Rightarrow (\beta \Rightarrow \alpha)$$

L2. $(\alpha \Rightarrow (\beta \Rightarrow \gamma)) \Rightarrow ((\alpha \Rightarrow \beta) \Rightarrow (\alpha \Rightarrow \beta))$
L3. $(\neg \beta \Rightarrow \neg \alpha) \Rightarrow ((\neg \beta \Rightarrow \alpha) \Rightarrow \beta)$
L4. $\forall x \alpha (x) \Rightarrow \alpha (x | t)$
L5. $\forall x (\alpha \Rightarrow \beta) \Rightarrow (\alpha \Rightarrow \forall x \beta)$
A1. $\forall x \Pi (x, x)$
A2. $\forall x \forall y \forall z \Pi(x, z) \land \Pi(y, z) \Rightarrow \Pi(x, z)$
T1. $\forall y \forall z \Pi(y, z) \Rightarrow \Pi(z, y)$
T2. $\Pi(x, y) \land \Pi(y, z) \Rightarrow \Pi(x, z)$
 \vdots

Fragments of reality described truly by this theory





For a given FORMAL THEORY

we can look for a domain which is adequately described

On the other hand For a given fragment of reality we wish to find (to create) appropriate formal theory Let us consider for instance a geometry of visual perception two stimuli are indistinguishable if the gap or difference between them is too small More formally:

two stimuli α and β are indistinguishable if for some function f holds

$$|f(\alpha) - f(\beta)| < \epsilon$$

The formal (SYNTACTIC) theory given by Roberts (1973)

A1. B(x, y, z) \Rightarrow B(z, y, x) A2. B(x, y, z) \lor B(x, z, y) \lor B(y, x, z) A3. B(x, y, u) \land B(y, z, u) $\land \neg$ B(x, y, z) \Rightarrow u I y \land u I z A4. \neg u I v \Rightarrow (B (x, u, v) \land B(u, v, y) \Rightarrow B(x, u, y)) A5. B(x, y, z) \land B(y, x, z) \Rightarrow x I y \lor (z I x \land z I y) A6. x I y \Rightarrow B(x, y, z)

One step further if instead

$$s(\boldsymbol{a}, \boldsymbol{b}) = \begin{cases} 0, & |f(\boldsymbol{a}) - f(\boldsymbol{b})| \ge \boldsymbol{e} \\ 1, & |f(\boldsymbol{a}) - f(\boldsymbol{b})| < \boldsymbol{e} \end{cases}$$

one wishes

$$s: U \times U \to [0, 1]$$

If one wishes to perform calculations with degrees of similarity or indistinguishability, then it turns out it is impossible to create a theory in a purely syntactic form. Admitting the gradually in conceiving the reality we must use fuzzy sets concepts as a formal tool (the best invented till now) to formulate theories in a SEMANTIC form

STOCHASTICS

A stochastic phenomenon is one whose observed data exhibit

chance regularity patterns

 (Ω, A, P)

Elementary school level

 $P: A \rightarrow [0, 1]$

Higher school level

$$X : \Omega \to R$$

 $X^{-1}(B) \in A$ for any Borel set B

characterised by two basic functions

 $F_{X}\left(x\right) \qquad \qquad f_{X}\left(x\right)$

and parameters known as moments

$$\alpha_{k} = \int x^{k} d F_{X}(x), \ k = 1, 2, ...$$

Observed regularities ore formulated in the form of LAWS

e.g. Gompertz Law of mortality, Makeham's law, Pareto law for income distribution, etc. etc.

University level

 \rightarrow Stochastic calculus

 \rightarrow Uncertainty modelling

two fundamental stones:

1. Theory of preferences for uncertain contingencies

(expected and non-expected utility theories)

2. Theory of contingent claims

ultimate goods are treated as contingent consumption claims i.e. entitlements

to commodities valid only under specified states of world