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Dynamics of Fuzzy Models for Market Players. The Three Companies Case

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Abstract. The analysis of a class of models for systems of companies on the market has been developed in a series of previous papers. Here, we model and analyze the dynamics of systems of companies selling the same product on the market for the case when the companies adopt two different price strategies, while conducting monitoring of the prices used by the other companies. The market players may use two different strategies in adapting the selling prices of their products. The decision making process for deciding the prices is modeled as a fuzzy decision making.

Keywords. Market model, fuzzy decision, dynamic behavior, strategy, vendor agility

1. Introduction

The behavior of small vendors in free market is a volatile dynamical process, involving both psychological and economical factors. The vendors may be conducted by the goal of maximizing profit, which is a reasonable purpose, yet other factors, such as the personal pride, the desire to overwhelm the competitors, and the desire to capture the customers of the competitors may play a role in the vendors' momentary strategy. Such intricate behaviors and the resulting apparently unexplained evolution of the prices are not uncommon situations in a market of small vendors and in a transitory economy, as seen in the fast developing countries. A typical case is that of the small vendors whose bases for commerce are mere "kiosks" or small networks of such street-based selling points. The result perceived by the buyers is a chaotic situation with unpredictable price changes and almost daily significant variations of the price, moreover unexpectedly high price differences between neighboring vendors. Such wild price fluctuations both in time and along a small space are stunning when regarded from the point of view of the classic economic theory, because no reasonable explanations seem to exist. Yet, anyone was able to experience the existence of such behaviors in the street-based markets in East European countries, for the last two decades.

The first goal of the reported research is to model, explain, analyze and predict the medium- and long-term behavior of vendors in small markets dominated by volatility and intricate, non-standard strategies. The evolution of the prices and of the profits of the vendors in the market is dominated by the strategy they adopt. We model the decision-making process using fuzzy logic. Moreover, in the implemented economic models, the decision-making process is based on feedback. Indeed, the decisions of the players are influenced in a direct manner by their strategy, moreover by the evolutions of the others companies in the market. The fluctuations of the prices registered on the market are reflected in the profits of the companies and consequently in the evolution of the selling prices. Actually, every company learns the price of the competitors with a certain delay, and the delays create favorable conditions for oscillating behavior. Because the whole loop <decision maker – prices used by the vendors – market feedback – competition response> may be non-linear, we may expect nonlinear dynamics occur in the evolution of the process. Notice that the decision making may be performed based on expert systems or decision support systems. In that case, we may regard the process and its nonlinear dynamics as induced by the machine, specifically, a machine based on fuzzy logic. This is an interesting problem by itself, namely the nonlinear dynamics induced by expert or decision-support machines in an economic process.

In microeconomic systems, uncertainty is always present. However, the statistic methods may not represent an adequate instrument for the analysis [2], [5]. Modeling these systems requires fuzzy logic, because the last allows the modeling of human reasoning, mainly the qualitative reasoning frequently used in everyday life and in decision-making.

A great deal of attention has been paid to applying A.I. techniques to the analysis of economic processes [3], [4], [10], [11], [25], [26]. Also, the theory of nonlinear dynamics (chaos theory) has been extensively applied to investigate economic processes that present intricate dynamics [12], [13], [15], [24]. In previous researches, we combined the theory of fuzzy logic and chaos theory to decision making and economic processes [6-8], [12-23]. A special attention has been paid to the role of the behavior of the players (actors) and the role of the delays in feedback decision-making loops in such processes [1], [9].

The nonlinearity of the fuzzy systems with defuzzification that model the behavior of the companies, combined with the feedback may produce an oscillatory behavior and nonlinear dynamics, which are correspondingly induced to the expert systems and decision support systems.

The practical questions we answer in this paper are: i) how the strategy of the players in a system of three vendors selling the same product in the market influence their profit, assuming that their decision making is based on fuzzy logic; ii) how stable is a system of vendors selling the same product, assuming their price strategy is stable and assuming some initial condition regarding the selling prices; iii) how different is the dynamics of a system of three vendors selling the same product when they use different strategies, use different initial prices, and adjust with different delays to the competition; iv) how sensitive is the average outcome and the range of the profit of the vendors to their initial choice of selling prices; v) how the delay in gathering information on the competition modifies the profit of the vendors after the transitory regime of their dynamical behavior ends.

In many respects, this paper also produces evidence on how a player vending a product in the market can act with agility to improve its performance. The agile player may be quicker in determining the behavior of the competition, or it may adopt a better selling strategy, or it may use more suitable initial prices. The agility of a company has many facets, and for sure the agility is a characteristic which has much in common with the dynamical behavior.

2. Modeling issues

2.1. Generalities and notations

We have studied the evolution of the prices and profits into a market where two companies sell the same product [22]. The decision-making process modeled in that research is influenced by the changes of the selling prices of the concurrence, which were perceived (received) by a certain delay. Then, we have introduced in the model two types of strategies of the players in the market, the first called *comp-benefit*, and the second *max-benefit* (see [20]). The class of economic models that we propose has supported several refinements in time. The refinements are aimed to allow us for modeling various possible situations.

The first type of strategy is a "selfish", "envy-based" strategy, where the policy of the company regarding the selling price is directly connected to the reactions of the concurrence. Companies using this strategy try to obtain a similar profit to that of the most profitable firm. In contrast, the companies using the *max-benefit* strategy aim to the maximization of the profit, regardless of the benefits registered by the concurrent firms. Such companies estimate the benefits/profits that they may obtain for a positive or negative fluctuation of the selling price and then they take the decision if the price will be maintained, increased or decreased. In our previous models, both companies utilized the same type of strategy. In either one of the strategies, the players adjust the prices in a specified amount, named *increment* (the increment may be, in fact, also negative.) The increment can be a constant or a fuzzy variable, the later being computed from the differences of the recorded benefits. Several results regarding the dynamics of such models have been published in the papers [20], [21], [22].

The strategies have been described and discussed in [20], [21]. In those papers, we have simulated systems with N players, all behaving according to the same strategy. In this paper, we report on modeling mixed systems, that is, systems with players using any of two different strategies. For the readers' convenience, we briefly recall the two strategies presented in references [20]. The sections 2.2, 2.3, and 2.5 in this paper have been reported in [20] and [22], where a few more details can be found. Some complementary results have been reported in [21].

Subsequently, we use the notations:

t	current time moment
N	number of players in the market
$\tau_{k,i}$	delay of learning by the player k of the price used by the competitor i. In general, $\tau_{k,i}$ is not related to $\tau_{i,k}$.
$p_k[t]$	price used by the vendor $\#k$ at time moment t
$b_{k,i}[t]$	estimation by the vendor $\#k$ of its profit at the next step in time, when it would compete only with the competitor $\#i$; the estimation is based on the information the vendor #*k* can obtain on the prices of the vendor #*i*, with the specified delay
$b_{med,k}[t]$	estimation by the player $\#k$ of its average profit at the next step in time, when it would compete with all the other players; the estimation is based on the information the vendor #*k* can obtain on the prices of the vendor #*i*, with the specified delays, and on averaging the values of $b_{k,i}[t]$ over all i
${}^{-}b_{k,i}$ $({}^{+}b_{k,i})$	estimation by the vendor $\#k$, as made by itself, of its profit at the next step in time, when it would compete only with the competitor $\#i$ and when it would adopt a price decrease (increase)
${}^{-}b_{med\,k}$ $({}^{+}b_{med\,k})$	estimation by the vendor $\#k$, as made by itself, of its average profit at the next step in time, when it would compete with all the other players, and when it would adopt a price decrease (increase)
$f(\cdot)$	denotes a function whose expression may be not completely defined at the moment of writing the general formula including $f(\cdot)$
$b_{i,k\ delayed}[t]$	estimation by the vendor $\#k$, as made by itself, of the profit of the competitor $\#i$, taking into account the delays in learning the prices of the concurrent companies
$b_{med\ delayed\ k}[t]$	estimation by the player $\#k$ of the average profit of the other players, taking into account the delays in learning the prices of the concurrent companies

Throughout the paper, we assume that the initial price used by every player is known and that all the vendors start their activity at the same moment, $t=0$. The initial price used by the vendor $\#k$ at the initial moment $t=0$ is denoted by $p_k[0]$. Moreover, we assume that until all players gather information about the other players, they do not change their prices. This assumption means that there is some initial period of time, equal to the maximum delay of learning the competitors' prices, when no competitor changes its price. Denoting by d_N the maximum delay in the system by $d_N = \max_{i,j}(\tau_{ij})$, the assumption means that $p_i[0] = p_i[1] = \cdots = p_i[d_N]$ for all i.

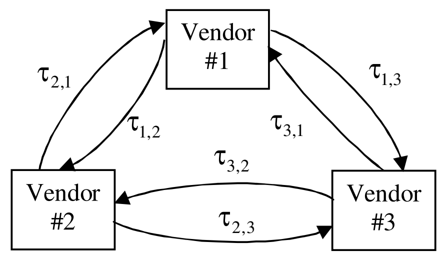


Fig. 1. Schematic diagram of the reduced market model

The schematic diagram of the market with three vendors is shown in Fig. 1. The market is equivalent to a ring-type network (graph) with edges labeled with the delays of acquiring information on the prices of the competitor. The information transfer in the market is characterized by the six delays, $\tau_{1,2}$ $\tau_{2,1}$ $\tau_{1,3}$ $\tau_{3,1}$ $\tau_{2,3}$ $\tau_{3,2}$.

2.2. Strategy for profit maximization "max-benefit" (partly after [20])

For every company, $\#k$, the obtained profit is estimated as a function of their actual price and of the most recent known price used by every competitor. The information on the price used by competitor $\#i$ is received with a specified delay, $\tau_{k,i}$. The information on the price used by the competitors is utilized in the purpose of choosing a suitable value for the vendor's selling price. Here, suitable means that that prices creates the premises to optimize the company's own selling profit, for example by increasing the selling volume.

First, the vendor $\#k$ computes the benefit it would have in case he would compete with a single competitor, the vendor $\#i$. In that case, the profit of the vendor $\#k$ in competition with the vendor $\#i$ would be

$$b_{k,1}[t] = f\left(p_k[t], p_1[t-\tau_{k,1}]\right), \ \ldots, \ b_{k,N}[t] = f\left(p_k[t], p_N[t-\tau_{k,N}]\right) \tag{1}$$

Here, f stands for some multivariable function, whose expression will be explained later. Then, the vendor $\#k$ computes the average profit of the other vendors. The expression of the average profit of the competition at time moment t is an average of all the other $N-1$ profits and is computed as:

$$b_{med\ k}[t] = \frac{\sum_{i=1, k\neq i}^{N} b_{k,i}[t]}{N-1}, \quad i \neq k\ . \tag{2}$$

Though the computation of the "profit equations" may look oversimplified because of the computation of the would-be profits as a set of profits in a one-to-one competition and by averaging of these individual results, the manner of reasoning resembles to the decisional process of humans when large amounts of data is available. Indeed, in such case, the humans tend to simplify the problem by splitting it to simpler problems and to create the whole problem result as an approximate solution based on averaging. In our case, the estimation of the profit in first determined in a one-to-one competition, applied for every opponent, and the overall estimation is only the average of particular results.

The next step, every vendor tries to determine his best next move in adjusting the price. He has the options to increase the price, to keep the price constant or to decrease the price. For all these options, he computes its next step estimated profit and then the vendor compares its possible outcomes. The vendor then selects the best choice it has, that is, the move which maximizes the profit at the next moment of time.

The estimation of profits ${}^{+}b_{k,i}$ that can be obtained at the subsequent moment of time $t+1$ (next step of inference and decision making), for an increase of the selling price, is realized assuming that the competitor prices are maintained at the same level:

$$ {}^{+}b_{k,1}[t] = f\left(p_k[t] + incr, p_1[t-\tau_{k,1}]\right), \ldots, {}^{+}b_{k,N}[t] = f\left(p_k[t] + incr, p_N[t-\tau_{k,N}]\right) \quad (3) $$

where *incr* denotes the increment of the price change.

The average profit is computed as:

$$ {}^{+}b_{med\ k}[t] = \frac{\sum_{i=1}^{N} {}^{+}b_{k,i}[t]}{N-1}, \quad i \neq k. \quad (4) $$

In a similar manner, the estimation of the profits ${}^{-}b_{k,i}$ of the firm $\#k$, if the current selling price is decreased, is determined by the formulas:

$$ {}^{-}b_{k,1}[t] = f\left(p_k[t] - incr, p_1[t-\tau_{k,1}]\right), \ldots, {}^{-}b_{k,N}[t] = f\left(p_k[t] - incr, p_N[t-\tau_{k,N}]\right) \quad (5) $$

$$ {}^{-}b_{med\ k}[t] = \frac{\sum_{i=1}^{N} {}^{-}b_{k,i}[t]}{N-1}, \quad i \neq k. \quad (6) $$

The vendor will change the price by incrementing or decrementing the current price, according to the case when he obtains the higher estimated profit. If none of the profits ${}^{+}b_{med\,k}$ and ${}^{-}b_{med\,k}$ is greater than the actual profit of the firm $\#k$, $b_{med\,k}$, then the decision taken by the company is to maintain the current price. According to the strategy "*max-benefit*", the purpose of every company using this strategy is to maximize its own profit, disregarding the other competitors' profits. This is a reasonable behavior of the companies; in this behavior, no subjectively made decision process interferes. In contrast, the strategy "*comp-benefit*", explained in Section 2.5, is an "envy-guided behavior", largely subjective.

2.3. The fuzzy increment (based on [20])

In case when the increment determination is modeled as a fuzzy decision, the market player needs to evaluate the benefits of each competing player $b_{i,k\ delayed}[t]$ and the average of the concurrence' profit, $b_{med\ delayed\ k}[t]$. We name $b_{med\ delayed\ k}[t]$ "delayed profit", because of the delaying in learning the competitors' prices.

$$b_{1,k\ delayed}[t] = f\left(p_1[t-\tau_{k,1}], p_k[t]\right), \dots,\ b_{N,k\,\mathrm{delayed}}[t] = f\left(p_N[t-\tau_{k,N}], p_k[t]\right) \tag{7}$$

$$b_{med\ delayed\ k}[t] = \frac{\sum_{i=1}^{N} b_{i,k\ delayed}[t]}{\mathrm{N - 1}}, \qquad i \neq k\,. \tag{8}$$

Here, $f(\cdot)$ is a function of the prices the vendors use at the previous moments. The expression of $f(\cdot)$ remains to be determined.

This "delayed profit" is needed in case of "*max-benefit*" strategy only for the computation of the increment value. As we explained, a firm which adopted this strategy does not take into account the profits of the other players for establishing their operation on the market.

A key factor in the strategy of the vendors is the way they modify the product price when in accordance with the profit they obtain and in accordance to the prices the competitors use. This is probably one of the factors most difficult to model and we propose a description of the price adaptation which is based on fuzzy logic. The reasons to use fuzzy logic in the model is that it probably better models the vendor decision-making, moreover a fuzzy logic description is easily understandable by and easily modified. Nonetheless, the fuzzy logic description has a counterpart representation as a real valued function, and we need to determine that function. Finding the function is the object of the subsequent sections.

The increment value used by the vendor #k, $incr_k[t]$, computed by a fuzzy method, is an expression of the difference between the profits of the current company and the estimated profit of the concurrence. This difference is subsequently denoted by dif_profit. For a large difference, the increment value will be large; conversely, for a small difference, the increment value is small. In Table 1 are described the inference fuzzy rules (Mamdani type), and in Fig. 2 are shown the membership functions, as used in the computation of the increment with a fuzzy method. The reason to use an upper limit (at value 2) for the universe of discourse is the need to define a real valued defuzzified value. Indeed, with the upper limit at infinity, the center of gravity defuzzification would yield an infinite value for some range of *x* and *y*.

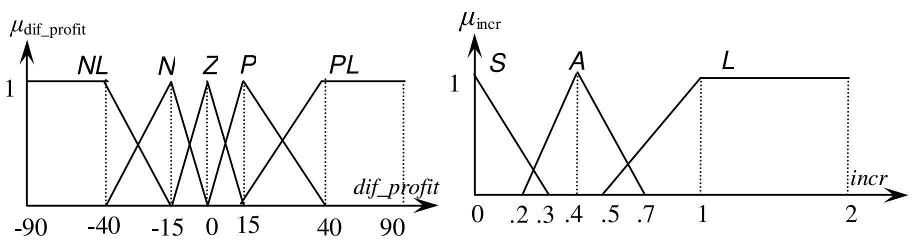


Fig. 2. Membership functions for the linguistic variables "*dif_profit*" and "*increment*" [22]

Table 1

Rules for computing the fuzzy increment using price difference [22]

dif profit	negative large (NL)	negative (N)	zero (Z)	positive (P)	positive large (PL)
increment	large (L)	average (A)	small (S)	average (A)	large (L)
Decrement rules	For decreasing the price three rules are used *: $p_k[t+1]=p_k[t]$-incr				
Increment rules			For increasing the price three rules are used *: $p_k[t+1]=p_k[t]$+incr		

* When ǀ *Dif_profit* ǀ < 1 then *incr* = 0 (exception case)

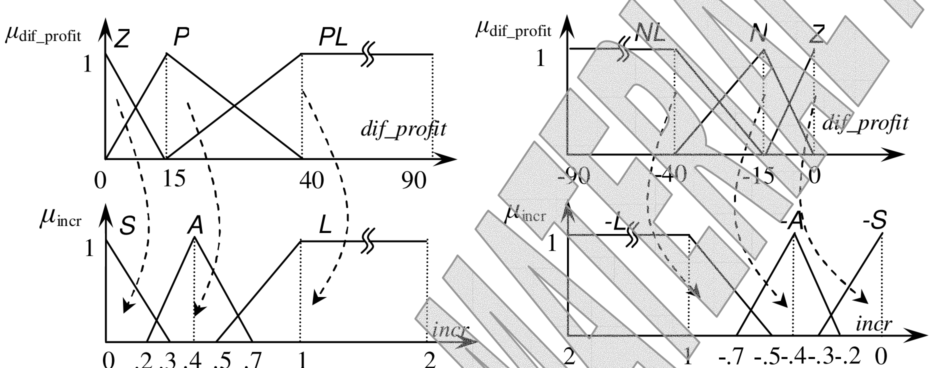


Fig. 3. Explanatory for the rule application

The choice of the membership function shown in Fig. 2 is partly arbitrary, partly based on common sense arguments in modeling the behavior of the vendors. The use of triangular membership functions instead of other type of membership functions, e.g., Gaussian membership functions, is justified by the ease of computation and by the good approximation the triangular functions offer to other intricate functions.

2.4. The characteristic function of the fuzzy increment model

The fuzzy system modeling the increment computation is determined by the rules in Table 1. The rules summarized in that table read, as rules:

R1 Rule #1: *If the differential profit is Negative large, then the increment is (negative) Large.*
R2 Rule #2: *If the differential profit is Negative, then the increment is (negative) Average.*
R3 Rule #3: *If the differential profit is Zero, then the increment is Small.*
R4 Rule #4: *If the differential profit is Positive, then the increment is (positive) Average.*
R5 Rule #5: *If the differential profit is Positive Large, then the increment is (positive) Large.*
R6 Rule #6: *If* | *Dif_profit* | < 1*, then the increment* = 0. (This rule supersedes rule R3).

The rules correspond to a Mamdani-type fuzzy system with a single input and a single output. The defuzzification by center of gravity is performed according to:

$$y_{cog} = \frac{\int_{-\infty}^{\infty} y \cdot \mu_{out}(y;x) \cdot dy}{\int_{-\infty}^{\infty} \mu_{out}(y;x) \cdot dy} \tag{9}$$

where y_{cog} is the center of gravity (cog) defuzzified value for $\mu_{out}(y;x)$, which is the overall output membership function with parameter x, that is, for a given input value x.

The input membership functions are

$$\mu_Z(x) = \begin{cases} \dfrac{x+15}{15} & x \in [-15,0] \\ 1 - \dfrac{x}{15} & x \in [0,15] \\ 0 & elsewhere \end{cases}, \quad \mu_P(x) = \begin{cases} \dfrac{x}{15} & x \in [0,15] \\ 1 - \dfrac{x-15}{25} & x \in [15,40] \\ 0 & elsewhere \end{cases}, \tag{10}$$

$$\mu_{PL}(x) = \begin{cases} \dfrac{x-15}{25} & x \in [15,40] \\ 1 & x \in [40,90] \\ 0 & elsewhere \end{cases}$$

The output membership functions are

$$\mu_S(y)=\begin{cases}1-\dfrac{y}{0.3} & y\in[0,0.3]\\ 0 & elsewhere\end{cases},\qquad \mu_A(y)=\begin{cases}\dfrac{y-0.2}{0.2} & y\in[0.2,0.4]\\ 1-\dfrac{y-0.4}{0.3} & y\in[0.4,0.7]\\ 0 & elsewhere\end{cases},\tag{11}$$

$$\mu_L(y)=\begin{cases}\dfrac{y-0.5}{0.5} & y\in[0.5,1]\\ 1 & y\in[1,2]\\ 0 & elsewhere\end{cases}.$$

Denote by x_0 some specified value of the input. When $x_0\in[0,15]$, the two active rules are R3 and R4. Consequently, the overall output membership function is:

$$\mu_{out}(y\,;x_0)=\begin{cases}\max\{\min[\mu_Z(x_0),\mu_S(y)],\min[\mu_P(x_0),\mu_A(y)]\} & y\in[0,\ 0.7]\\ 0 & y\in[0.7,2]\end{cases}\tag{12}$$

Therefore, the input-output function is:

$$y_{cog}(x_0)=\frac{\int_{-\infty}^{0}y\cdot 0\cdot dy+\int_{0}^{0.7}y\cdot\max\{\min[\mu_Z(x_0),\mu_S(y)],\min[\mu_P(x_0),\mu_A(y)]\}\cdot dy+0}{\int_{-\infty}^{\infty}\mu_{out}(y;x_0)\cdot dy}\tag{13}$$

Notice that the overall input-output function passes through the points (0,0); (15, 0.4), and (40, 1). For $x\geq 0.7$, the function is a constant, valued $cog(L)=\dfrac{0.25\cdot 0.83+1.5\cdot 1}{1.25}=$ 0.1367. The qualitative graph of the input-output function is shown in Fig. 4.

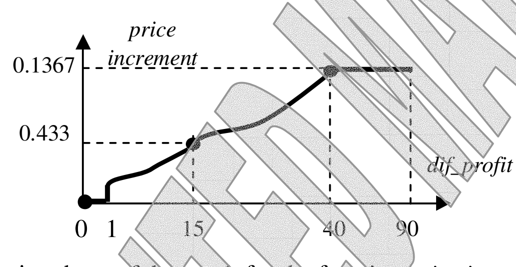


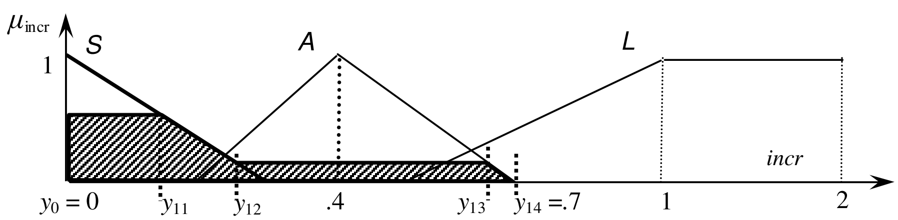
Fig. 4. Qualitative shape of the graph for the function *price increment* vs. *dif_profit*. The marked point are precise points through which the function goes

Proposition. The increment vs. differential profit function of the model is a piecewise rational function, with the denominator a cubic in the input value, and the nominator a quadratic in the input value.

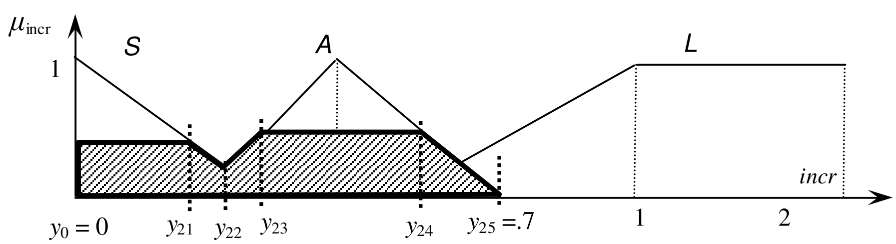
To compute the function (14), we need a few intermediate computations. The intersection of the graphs of the membership functions S and A is in the point defined by the equation

$$1-y_0/0.3=(y_0-0.2)/0.2 \;\Rightarrow y_0=0.24\,,\; \mu(y_0)=(0.24-0.2)/0.2=0.2$$

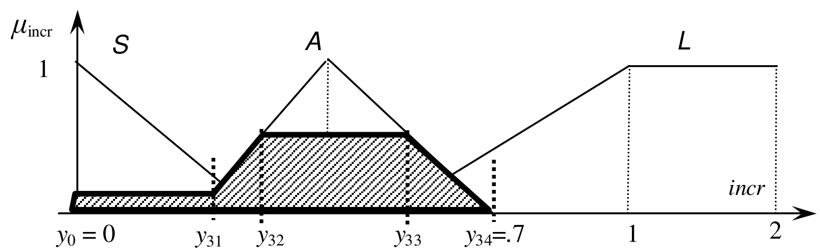
The corresponding membership input functions take the value 0.2 respectively for $\mu_Z(x_0)=0.2 \Rightarrow 1-x_0/15=0.2 \Rightarrow x_0=12$ and $\mu_P(x_0)=0.2 \Rightarrow x_0/15=0.2 \Rightarrow x_0=3$.



(a) $1 \le x_0 \le 3$, $\mu_P(x_0) \le 0.2$



(b) $3 \le x_0 \le 12$, $\mu_Z(x_0) \ge 0.2$ and $\mu_P(x_0) \ge 0.2$



(c) $x_0 \ge 12$, $\mu_Z(x_0) \le 0.2$

Fig. 5. The type of output membership function, depending on the input value

According to the case $x_0 \in [1,3]$, $x_0 \in [3,12]$, or $x_0 \in [12,15]$, the shape of the output membership function is as in Fig. 5. Precisely,

A) When $\mu_Z(x_0) \geq 0.2$, and $\mu_P(x_0) \leq 0.2$, i.e., $1 \leq x_0 \leq 3$, the graph of the output membership function is as in Fig. 5 (a).
B) When $\mu_Z(x_0) \geq 0.2$ and $\mu_P(x_0) \geq 0.2$, i.e., $3 \leq x_0 \leq 12$, the graph of the output membership function is as in Fig. 5 (b).
C) When $\mu_Z(x_0) \leq 0.2$, i.e., $x_0 \geq 12$, the graph of the output membership function is as in Fig. 5 (c).

Notice that $\mu_Z(x_0) \leq 0.2$ implies $\mu_P(x_0) \geq 0.8$ and $\mu_P(x_0) \leq 0.2$ implies $\mu_Z(x_0) \geq 0.2$, because, according to the particular choice of the two membership functions here, $\mu_Z(x_0) = 1 - \mu_P(x_0)$.

Thus, the integral in equation (14) should be computed according to the cases in Fig. 5, and finally it depends on the input value x_0. For the case in Fig. 5 (a):

$$y_{cog}(x_0) = \frac{\int_{y_0}^{y_{11}} y \cdot \mu_Z(x_0) \cdot dy + \int_{y_{11}}^{y_{12}} y \cdot \mu_S(y) \cdot dy + \int_{y_{12}}^{y_{13}} y \cdot \mu_P(x_0) \cdot dy + \int_{y_{13}}^{y_{14}} y \cdot \mu_A(y) \cdot dy}{\int_{y_0}^{y_{11}} \mu_Z(x_0) \cdot dy + \int_{y_{11}}^{y_{12}} \mu_S(y) \cdot dy + \int_{y_{12}}^{y_{13}} \mu_P(x_0) \cdot dy + \int_{y_{13}}^{y_{14}} \mu_A(y) \cdot dy} \quad (14)$$

For the case in Fig. 5 (b):

$$y_{cog}(x_0) = \frac{\int_{y_0}^{y_{21}} y \cdot \mu_Z(x_0) \cdot dy + \int_{y_{21}}^{y_{22}} y \cdot \mu_S(y) \cdot dy + \int_{y_{22}}^{y_{23}} y \cdot \mu_A(y) \cdot dy + \cdots}{\int_{y_0}^{y_{21}} \mu_Z(x_0) \cdot dy + \int_{y_{21}}^{y_{22}} \mu_S(y) \cdot dy + \int_{y_{22}}^{y_{23}} \mu_A(y) \cdot dy + \cdots}$$
$$\frac{\cdots + \int_{y_{23}}^{y_{24}} y \cdot \mu_P(x_0) \cdot dy + \int_{y_{24}}^{y_{25}} y \cdot \mu_A(y) \cdot dy}{\cdots + \int_{y_{23}}^{y_{24}} \mu_P(x_0) \cdot dy + \int_{y_{24}}^{y_{25}} \mu_A(y) \cdot dy} \quad (15)$$

For the case in Fig. 5 (c):

$$y_{cog}(x_0) = \frac{\int_{y_0}^{y_{31}} y \cdot \mu_Z(x_0) \cdot dy + \int_{y_{31}}^{y_{32}} y \cdot \mu_A(y) \cdot dy + \int_{y_{32}}^{y_{33}} y \cdot \mu_P(x_0) \cdot dy + \int_{y_{33}}^{y_{34}} y \cdot \mu_A(y) \cdot dy}{\int_{y_0}^{y_{31}} \mu_Z(x_0) \cdot dy + \int_{y_{31}}^{y_{32}} \mu_A(y) \cdot dy + \int_{y_{32}}^{y_{33}} \mu_P(x_0) \cdot dy + \int_{y_{33}}^{y_{34}} \mu_A(y) \cdot dy} \quad (16)$$

The values of the interval boundaries $y_{\alpha\beta}$ and the corresponding output membership function are computed as follows, for the case in Fig. 5 (a), $x_0 \in [1,3]$.

- $\mu_{out}(y) = \mu_Z(x_0)$ for $y_0 = 0 \le y \le y_{11}$, where $\mu_S(y_{11}) = \mu_Z(x_0)$, thus $(0.3 - y_{11})/0.3 = \mu_Z(x_0)$, or
  $y_{11} = 0.3 - 0.3 \cdot \mu_Z(x_0) = 0.3 - 0.3 \cdot (15 - x_0)/15 = 0.3x_0/15$.
- $\mu_{out}(y) = (0.3 - y)/0.3$ for $y_{11} \le y \le y_{12}$, where $\mu_S(y_{12}) = \mu_P(x_0)$. After computations, $y_{12} = (4.5 - 0.3x_0)/15$.
- $\mu_{out}(y) = \mu_P(x_0)$ for $y_{12} \le y \le y_{13}$, where $\mu_A(y_{13}) = \mu_P(x_0)$, $y_{13} > 0.4$. $\mu_S(y_{12}) = \mu_P(x_0)$. After computations, $y_{13} = 0.7 - 0.3 \cdot x_0/15$.
- Finally, $\mu_{out}(y) = \mu_A(y)$ for $y_{13} \le y \le y_{14}$.

The output value (14) becomes:

$$y_{cog}(x_0) = \frac{\int_{y_0}^{0.3 \cdot x_0/15} y \cdot \mu_Z(x_0) \cdot dy + \int_{0.3 \cdot x_0/15}^{(4.5-0.3x_0)/15} y \cdot \mu_S(y) \cdot dy + \cdots}{\int_{y_0}^{0.3 \cdot x_0/15} \mu_Z(x_0) \cdot dy + \int_{0.3 \cdot x_0/15}^{(4.5-0.3x_0)/15} \mu_S(y) \cdot dy + \cdots}$$

$$\frac{\cdots + \int_{(4.5-0.3x_0)/15}^{0.7-0.3 \cdot x_0/15} y \cdot \mu_P(x_0) \cdot dy + \int_{0.7-0.3 \cdot x_0/15}^{0.7} y \cdot \mu_A(y) \cdot dy}{\cdots + \int_{(4.5-0.3x_0)/15}^{0.7-0.3 \cdot x_0/15} \mu_P(x_0) \cdot dy + \int_{0.7-0.3 \cdot x_0/15}^{0.7} \mu_A(y) \cdot dy}$$

Replacing now the membership functions, we obtain

$$y_{cog}(x_0) = \frac{\int_{y_0}^{0.3 \cdot x_0/15} y \cdot \frac{15 - x_0}{15} \cdot dy + \int_{0.3 \cdot x_0/15}^{(4.5-0.3x_0)/15} y \cdot \frac{0.3 - y}{0.3} \cdot dy + \cdots}{\int_{y_0}^{0.3 \cdot x_0/15} \frac{15 - x_0}{15} \cdot dy + \int_{0.3 \cdot x_0/15}^{(4.5-0.3x_0)/15} \frac{0.3 - y}{0.3} \cdot dy + \cdots}$$

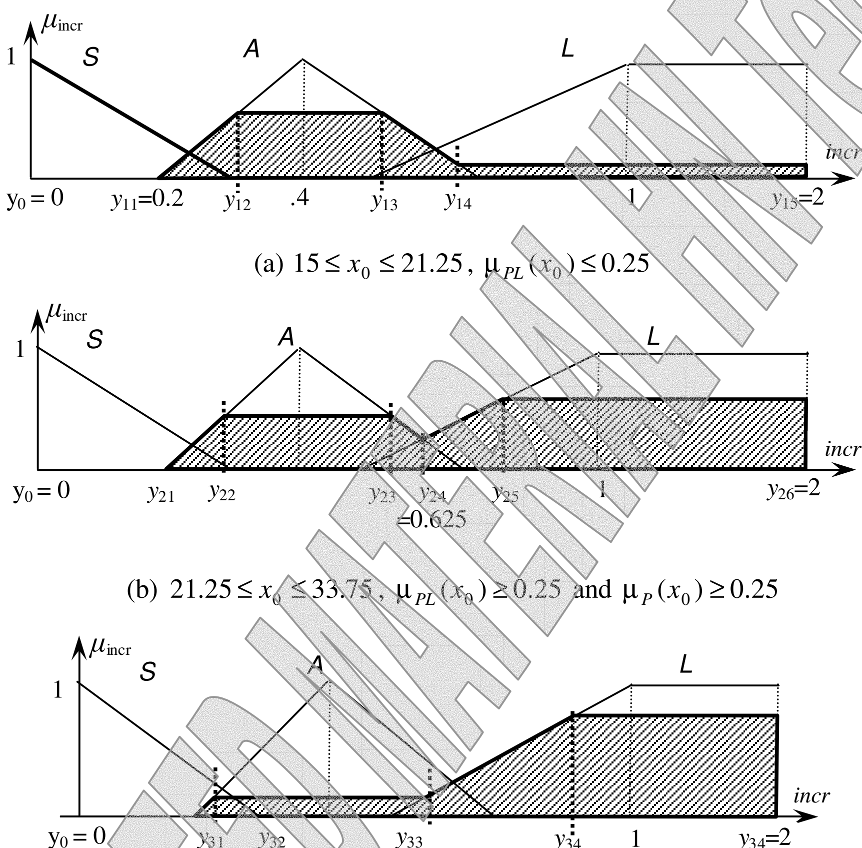
$$\frac{\cdots + \int_{(4.5-0.3x_0)/15}^{0.7-0.3 \cdot x_0/15} y \cdot \frac{x_0}{15} \cdot dy + \int_{0.7-0.3 \cdot x_0/15}^{0.7} y \cdot \frac{0.7 - y}{0.3} \cdot dy}{\cdots + \int_{(4.5-0.3x_0)/15}^{0.7-0.3 \cdot x_0/15} \frac{x_0}{15} \cdot dy + \int_{0.7-0.3 \cdot x_0/15}^{0.7} \frac{0.7 - y}{0.3} \cdot dy} \tag{17}$$

Notice that the integrals at the denominator produce at most the cube of *y*, while the integrals at the nominator produce at most the square of *y*. Therefore, after replacing the limits of integration, we shall obtain a rational function in x_0, with a cubic denominator in x_0 and a square nominator in x_0. The computations in (17) are straightforward; the result is

$$y_{cog}(x_0) = \frac{0.01 \cdot x_0^{\,2} \cdot (15 - x_0)/15^2 + 0.3^2 \cdot (1 - 6 \cdot x_0^{\,2}/15^2 + 4 \cdot x_0^{\,3}/15^3)/6 + \cdots}{0.3 \cdot x_0 (15 - x_0)/15^2 + 0.01 \cdot (15 - 2x_0) + \cdots} \qquad (18)$$

$$\frac{\cdots + 0.2 \cdot x_0 \cdot (15 - 0.6 \cdot x_0)/15^2 + (0.7 - 0.01 \cdot x_0) \cdot (0.7 - 0.2 \cdot x_0 / 15) \cdot x_0 / 15}{\cdots + 0.4 \cdot x_0 / 15 + 0.15 \cdot x_0^{\,2} / 15}$$

In a similar manner one computes the characteristic function for the cases (b) and (c). The cases for computing the increment function for $x_0 \in [15, 40]$ are shown in Fig. 6.



(a) $15 \le x_0 \le 21.25$, $\mu_{PL}(x_0) \le 0.25$

(b) $21.25 \le x_0 \le 33.75$, $\mu_{PL}(x_0) \ge 0.25$ and $\mu_P(x_0) \ge 0.25$

(c) $x_0 \ge 33.75$, $\mu_P(x_0) \le 0.25$

Fig. 6. The type of output membership function, depending on the input value

Notice that the maximum allowed increment, here having the value 2, plays an important role in the shape of the characteristic function; indeed, when the maximum increment varies, so varies the center of gravity of the section of the output membership function produced by the *L* membership function.

2.5. "Comparative-benefit" strategy [20]

Te comparative benefit strategy is based on comparisons of the own benefit with the benefits of the other firms. The evolution on the market for a firm that uses the strategy based to "envy-guided behavior" is different in comparison to the behavior described by the first strategy. In this case, if the profit of the firm, $b_{med\,k}$, is smaller than the "delayed profit" of the concurrence $b_{med\ delayed\ k}[t]$, then we compare the price practiced by the current company $\#k$ with the average price of the competitors $p_{med\,k}$. If the current price is smaller (respectively is bigger), then the price of the company will be adjusted to make the price closer to the average price, in the hope that, in this way, the future profits will be similar to that of the most profitable companies.

$$p_{med\ k}[t] = \frac{\sum_{i=1}^{N} p_i[t-\tau_{k,i}]}{N-1}, \quad i \neq k. \tag{19}$$

The algorithm applied in the simulation of this model is:

1. Initialize the lists of prices for the *N* companies. Initialize the fixed increment or chose a fuzzy increment. Initialize the number of time steps, *P*, $p \leftarrow P$ to perform the computation.
2. While ($p \geq 1$) do
3. for $k = 1$ to N, sequentially select each of the N companies and determine the average profit, as well as the own profit of the current firm at time moment *t*.
4. Estimate, based on "delayed prices" (prices learned with delay), the profits obtained by the concurrent firms,
5. $b_{med\ k}[t]$ and $b_{med\ delayed\ k}[t]$
6. Modify the prices applying the strategy according to the rules R1, R2, R3, using the *Procedure_price*.
7. $p \leftarrow p-1$, return to step 3

Procedure_price (company #k)

$p_k[t+1] = f\,(p_{med\ k}[t]\ ,\ b_{med\ k}[t], b_{med\ delayed\ k}[t])$

if $b_{med\ k}[t] < b_{med\ delayed\ k}[t]$ // profit lower than that of the concurrent firms

then if $p_k[t] \leq p_{med\,k}[t]$

then $p_k[t+1] = p_k[t] + incr$

else $p_k[t+1] = p_k[t] - incr$

if $b_{med\ k}[t] \geq b_{med\ delayed\ k}[t]$ // profit higher than that of the concurrent firms
then compute the profits $b_{med\ k}[t+]$ and $b_{med\ k}[t-]$, and determine the
price $p_k[t+1]$ according to $\max\{b_{med\ k}[t],\ b_{med\ k}[t+],\ b_{med\ k}[t-]\}$.

$$If\ \ \max\left\{b_{med\ k}[t],\ {}^{+}b_{med\ k}[t],\ {}^{-}b_{med\ k}[t]\right\} = \begin{cases} b_{med\ k}[t] \\ {}^{+}b_{med\ k}[t] \\ {}^{-}b_{med\ k}[t] \end{cases},\quad then\ \ \begin{cases} p_k[t+1] = p_k[t] \\ p_k[t+1] = p_k[t] + incr \\ p_k[t+1] = p_k[t] - incr \end{cases}$$

end_procedure

The profits are computed using a Mamdani-type set of rules with rule premises composed of two elementary premises. The corresponding fuzzy system has two input linguistic variables and a single output variable. The input variables, x_1 and x_2, are the current price used by the company under focus and the price used by a concurrent company (the second being known with some specified delay). The fuzzy output variable is the profit y of the company under discussion. Recall that single-input single-output (SISO) rules are used in the computation of the fuzzy increment, as described in Section 2.4.

The systems use the same rules as reported in [22]. For convenience, the rule-table is shown in (Table 2). However, notice that the strategy differs here, as explained. The simulations of these models have been made using an application developed in FuzzyCLIPS™6.1. FuzzyCLIPS™ is a programming language specifically designed for rule-based fuzzy reasoning and it is freely accessible [27]. The modifiers are the same as in FuzzyCLIPS, see [27] pp. 38-45. The choices of the number of players, of the strategies used by the players, of the delays, and of the type of increment are set in an initialization file of the software application. A version of the application can be tested freely on the web, at the address *http://www.etc.tuiasi.ro/sibm/Discipline/Sistem economic fuzzy.html*.

Table 2

Rules for determining the profit as a function with variables the price P1 used by the focused company, and the price of the concurrent P2. The notations are: VS – very small, S – small, M – average, H – high, and VH – very high [22]

		P2		
		Small	Average	High
P1	Small	M	More or less M	VH
	Average	S	H	Somewhat H
	High	VS	S	S

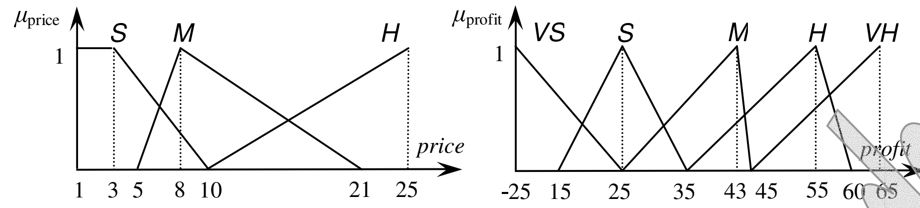


Fig. 7. Membership functions of the linguistic variables "*price*" and "*profit*" [22]

The triangular membership functions for price and profit (see Fig. 7) are represented in short-hand notations as (a,b,c), where a, b, and c represent the abscissas of the three vertices of the triangle, in ascending order. The general formula for these membership functions is

$$\mu(v) = \begin{cases} (v-a)/(b-a) & a \le v < b \\ (c-v)/(c-b) & b \le v \le c \\ 0 & elsewhere \end{cases} .$$

In case of trapezoidal membership functions, the shorthand notation includes four abscissas, (a,b,c,d), two of them being equal for rectangular trapezoids. For example, the membership functions of the price can be represented as $(1,1,3,10)$, $(5,8,21)$, and $(10,25,25)$.

The rules summarized in Table 2 read:

R1 Rule 1 ***If*** *the price of the vendor,* P1*, is small and the price of the competitor,* P2*, is small,* ***then*** *the profit of the vendor is average.*

R2 Rule 2 ***If*** *the price of the vendor,* P1*, is small and the price of the competitor,* P2*, is average,* ***then*** *the profit of the vendor is more or less average.*

R3 Rule 3 ***If*** *the price of the vendor,* P1*, is small and the price of the competitor,* P2*, is high,* ***then*** *the profit of the vendor is very high.*

R4 Rule 4 ***If*** *the price of the vendor,* P1*, is average and the price of the competitor,* P2*, is small,* ***then*** *the profit of the vendor is small.*

R5 Rule 5 ***If*** *the price of the vendor,* P1*, is average and the price of the competitor,* P2*, is average,* ***then*** *the profit of the vendor is high.*

R6 Rule 6 ***If*** *the price of the vendor,* P1*, is average and the price of the competitor,* P2*, is high,* ***then*** *the profit of the vendor is somewhat high.*

R7 Rule 7 ***If*** *the price of the vendor,* P1*, is high and the price of the competitor,* P2*, is small,* ***then*** *the profit of the vendor is very small.*

R8 Rule 8 ***If*** *the price of the vendor,* P1*, is high and the price of the competitor,* P2*, is average,* ***then*** *the profit of the vendor is small.*

R9 Rule 9 ***If*** *the price of the vendor,* P1*, is high and the price of the competitor,* P2*, is high,* ***then*** *the profit of the vendor is small.*

The choice of the rules is based on common sense reasoning and on economic theory. It is reasonable to assume that the vendor using smaller prices for the same sold product will have more customers and thus more profit. The numerical values used in the definition of the membership functions are arbitrary and are used for exemplification purpose only. The range of the prices in this example is extreme – prices vary in a ratio 1 to 25, which is rarely true. The lowest prices is 1 price unity, with the unity arbitrary (1 can represent 1 Euro or 125 Euros, for example). The unity corresponds here to the lowest price the vendors can practice to make profit according to the rules in Table 2. Therefore, the unit price should be substantially higher than the price the vendors pay themselves for the product.

2.6. Discussion of the models for the strategies

The membership functions used in the simulations are the same as in [21], for comparison purpose. The reader can find the mathematical expressions for the membership functions in [21]. The graphs of the membership functions used in [21] and here are shown in Fig. 7. The choice of the membership functions is largely arbitrary; other numerical values, and even other shapes of the membership functions (e.g., trapezoidal) may be used, according to the modeling purpose and the application in hand. With the choices for the membership functions and the rules presented above, the input-output characteristic of the fuzzy model is obtained as in Fig. 8. This figure shows, for convenience, two views of the same input-output function, to evidence that the function is non-monotonic, with a local maximum indicated by the arrow in Fig. 8. We emphasize that this local extremum is the key in inducing nonlinear oscillating behaviors of the system.

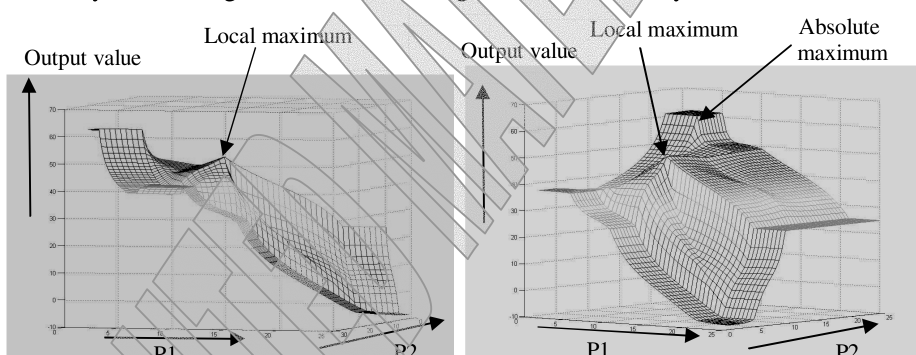


Fig. 8. Two 3D views of the input-output function of the fuzzy system with defuzzification, for the price of current firm P1 and the price of concurrence P2

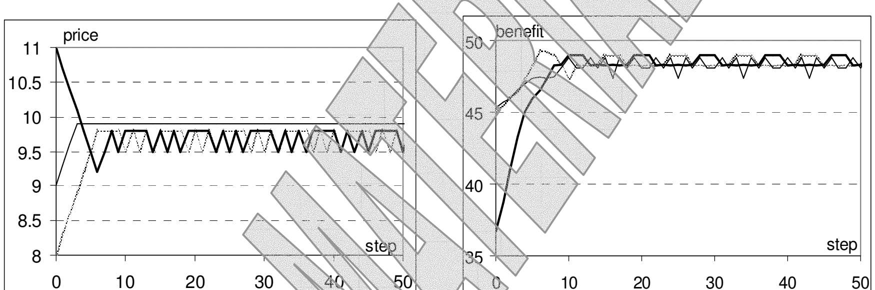
Notice that the function $f : R^2 \to R$ representing the characteristic function of the defuzzified system defined by the rules in Table 2 and the membership function in Fig. 8 goes through the points in the space R^3 standing for (P_1, P_2, *Profit*): (1,1,43), (1,3,43), (1,8,43), (3,8,43), (1,25,58.3), (8,3,25), (8,8,55), (8,25, 55), …

The local maximum of the two-input one-output characteristic in Fig. 8 produces the non-monotony of the characteristic. This non-monotony is due to the specific choice of the membership functions and of the rules. By changing any of these elements, one or several maxima can be produced, or the characteristic may be set to be monotonic.

3. Simulation results

In this section, we use the notation M for the strategy profit maximization ("max-benefit"), and the notation C for the "comparative-benefit" strategy. For example, the notation CCC means that all the vendors use the strategy "comparative-benefit", while the notation MCM means that the vendors #1 and #3 use the "max-benefit" strategy, while the vendor #2 uses the "comparative-benefit" strategy.

The simulations have been aimed to determine, in the first place, the dynamic behaviors of the network of three vendors when the strategies used are modified, moreover when the type of increment (crisp or fuzzy) is changed. In an example, we also determine the influence of the delays on the dynamic evolution. In all simulations, the initial prices of the product, practiced by the companies, are 8,9,11. In all but one cases, the delay matrix is [[0 3 4][2 0 2][3 3 0]].



(a) Price variation (case CMC, increment 0.3) (b) Profit variation (case CMC, increment 0.3)

Fig. 9. The evolution of the prices and the profits of three competitors

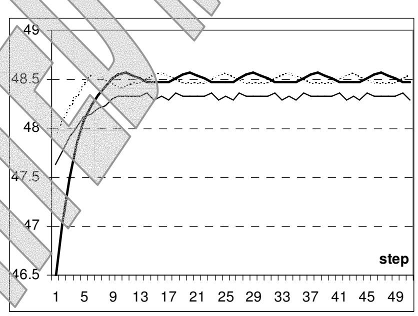


Fig. 10. The average benefit for firms in a window of dimension equal with 20 values

The graphics in Fig. 9 are obtained by simulation for the case when the companies use the strategies CMC and the fixed (crisp) increment is 0.3. After 7 steps, the systems enter in a loop of period 9 (see Fig. 9). A synthetic indicator of the overall network behavior is the average benefit of the companies. This indicator obviates the global periodic behavior and its evolution is shown in Fig. 10.

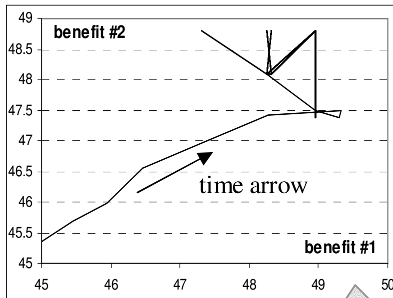
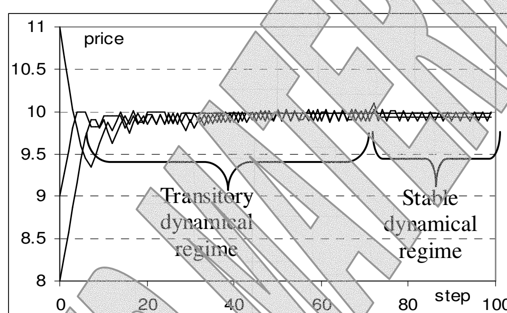
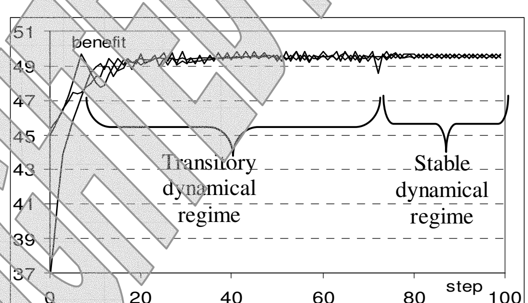


Fig. 11. Benefit co-evolution of the firm #1 and the firm #2

For the same input data and the same strategies, but using the fuzzy type increment, the system enters in a loop of period 2 after 85 steps. Notice in Fig. 12 (a) that the system transitory regime is significantly longer.



(a) Price variation (case CMC, fuzzy increment)



(b) Benefit variation (case CMC, fuzzy increment)

Fig. 12. The evolution of the prices and the profits of three competitors

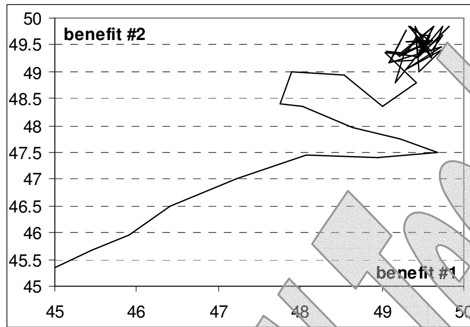
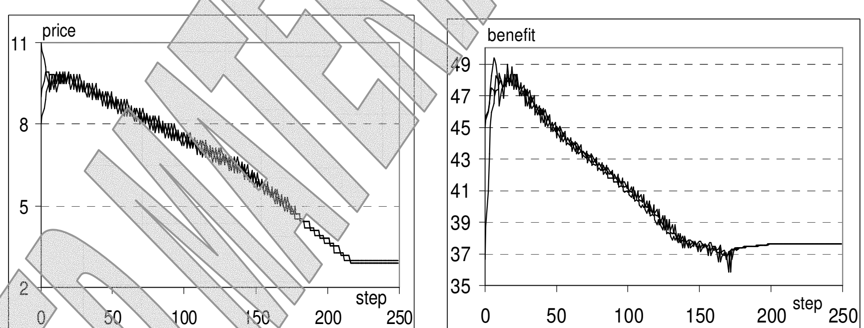


Fig. 13. Benefit co-evolution of the firm #1 and the firm #2

To clarify the way the systems correlate during the evolution of the network, we use throughout this paper diagrams of co-evolution. Such diagrams are phase diagrams for the system and the plots show if the benefits and prices of different vendors in the system evolve in a similar or dissimilar way. In Fig. 13, the benefit co-evolution for two vendors is plot.

Using the same initial prices practiced by the companies as in the previous cases, 8,9,11, the same matrix of delays [[0 3 4][2 0 2][3 3 0]], a fixed increment 0.3, but using the strategies CCC, the simulations show that the behavior of the network of three vendors stabilizes after 220 steps (see Fig. 14).



(a) Price variation (case CCC, increment 0.3) (b) Profit variation (case CCC, increment 0.3)

Fig. 14. The evolution of the prices and the profits of three competitors

The co-evolution of the prices and benefits for two vendors are shown in Fig. 15 and Fig. 16 respectively. Notice the time arrow indicating he sense of evolution and the fact that both the prices and the benefits decrease for both companies. This decrease is attributed to the type of strategy used. One can say that the envy-based strategy is detrimental for the companies, but is very beneficial to the customers, as the prices sharply decrease.

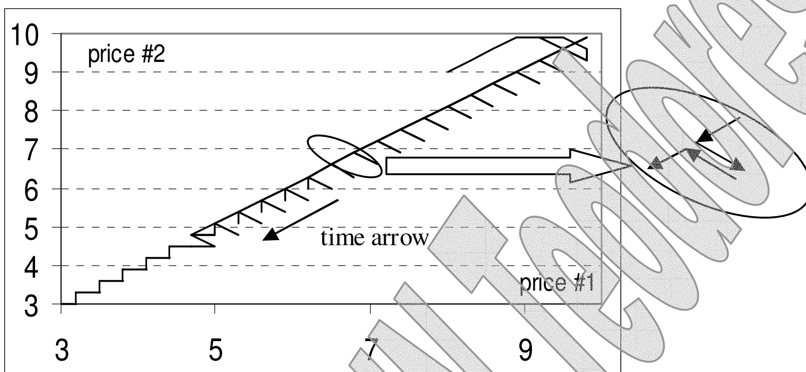


Fig. 15. Price co-evolution of the firm #1 and the firm #2

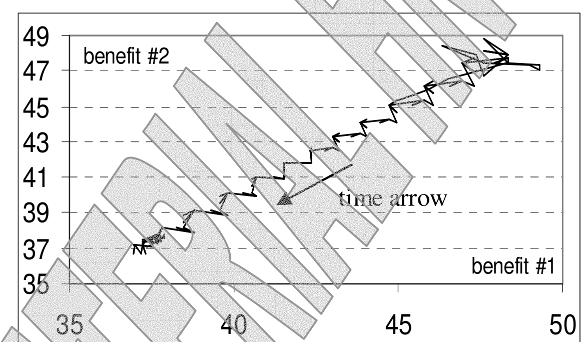
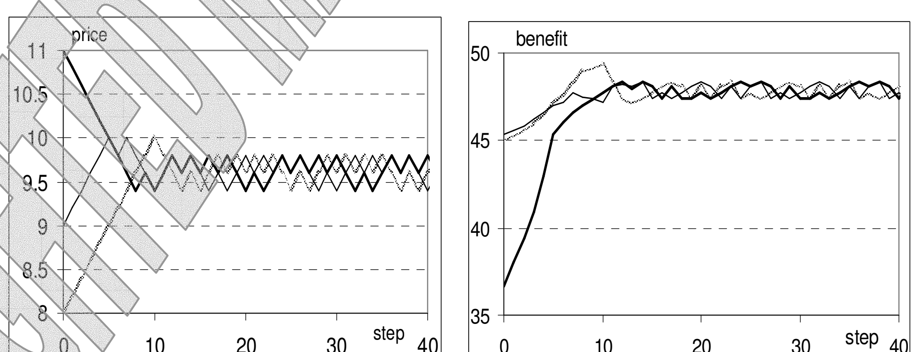


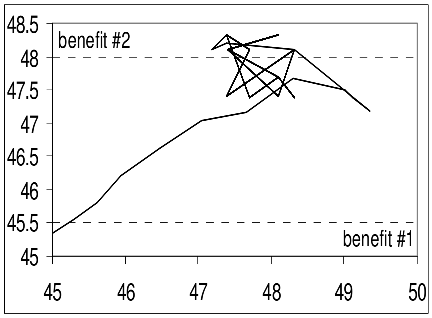
Fig. 16. Benefit co-evolution of the firm #1 and the firm #2

In another simulation, we kept the same conditions as above, except the fixed crisp increment is changed to 0.2. The dynamics is shown in Fig. 17. In this case the system enters in a loop of period 12 after 14 steps (see Fig. 18 (b)).

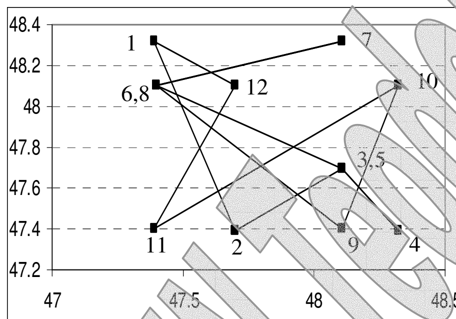


(a) Price variation (case CCC, increment 0.2) (b) Profit variation (case CCC, increment 0.2)

Fig. 17. The evolution of the prices and the profits of three competitors



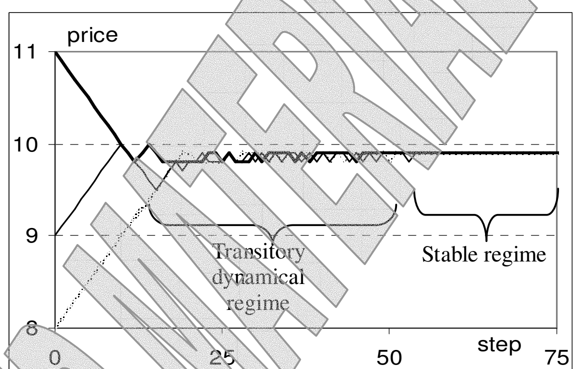
(a)



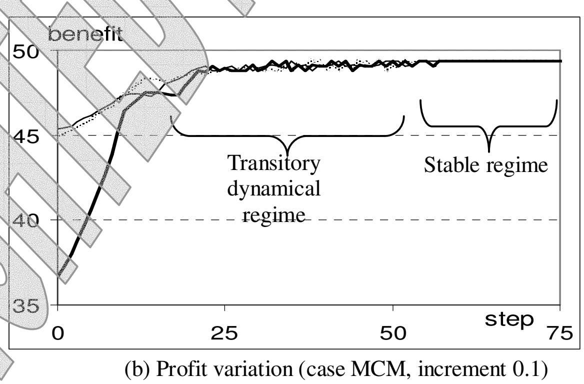
(b)

Fig. 18. (a) Benefit co-evolution of the firm #1 and the firm #2; (b) details of the loop

Yet in another simulation, we tested the MCM strategy with a fixed increment of value 0.1. In this case, the systems' evolution stabilizes after 57 steps; the transitory regime is quite long – see Fig. 19. The co-evolution graphs are shown in Fig. 20 and 21.

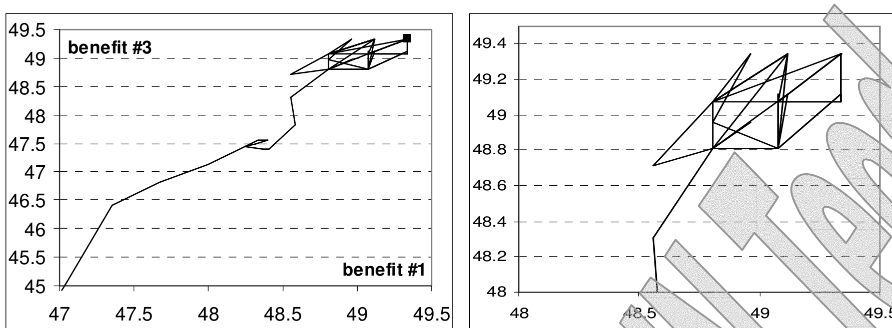


(a) Price variation (case MCM, increment 0.1)



(b) Profit variation (case MCM, increment 0.1)

Fig. 19. The evolution of the prices and the profits of three competitors



(a) (b)

Fig. 20. (a) Benefit co-evolution of the firm #1 and the firm #3; (b) details of the loop

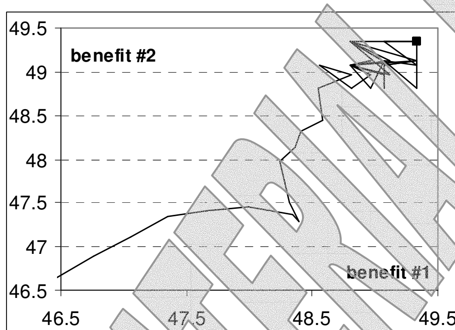
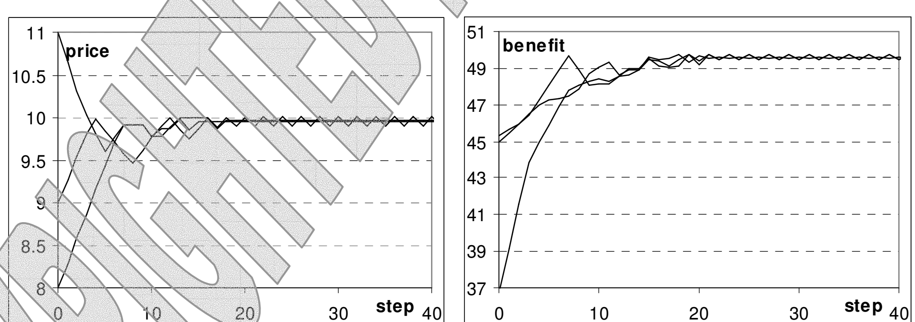


Fig. 21. Benefit co-evolution of the firm #1 and the firm #2

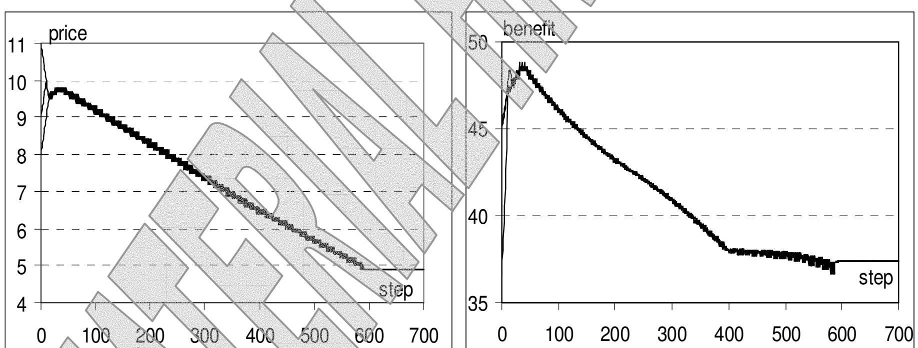
The system whose dynamics is plot in Fig. 22 corresponds to the system with evolution depicted in Fig. 19. Both systems have the same input data. The only difference is that the increment in the latter case is fuzzy. Notice that the network with systems using a fuzzy increment has a dynamics with a loop of period 2 and its transitory regime lasts only 19 steps (see Fig. 22).



(a) Price variation (case MCM, fuzzy increment) (b) Profit variation (case MCM, fuzzy increment)

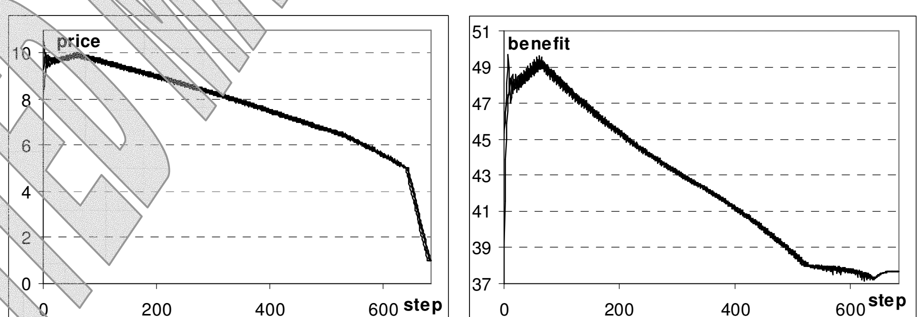
Fig. 22. The evolution of the prices and the profits of three competitors

The systems corresponding to the graphics in Fig. 23 start with the same initial prices as in the previous simulations (the companies use the prices 8,9, and 11), moreover have the same matrix of delays [[0 3 4][2 0 2][3 3 0]], and the same fixed increment 0.1. The difference between this case and the previous cases is that the vendors use a different set of strategies: the vendors #1 and #3 use the strategy M (profit maximization), while the vendor #2 uses the C strategy (comparison-based strategy). In case of the CCC method, the system is stable after 595 steps (the transitory regime is extremely long – see Fig. 23). When the increment is fuzzy (see Fig. 24), all the prices reach the value 1 after 685 steps. Notice that this is the inferior limit domain for the fuzzy variable price and consequently the simulation automatically stops.



(a) Price variation (case CCC, increment 0.1) (b) Profit variation (case CCC, increment 0.1)

Fig. 23. The evolution of the prices and the profits of three competitors



(a) Price variation (case CCC, fuzzy increment) (b) Profit variation (case CCC, fuzzy increment)

Fig. 24. The evolution of the prices and the profits of three competitors

The co-evolutions of the prices and benefits for the vendors #1 and #2 are represented in Fig. 25 and Fig. 26, respectively.

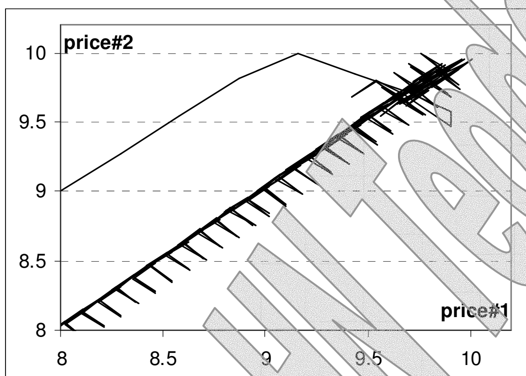


Fig. 25. Price co-evolution of the firm #1 and the firm #2 (case CCC, fuzzy increment)

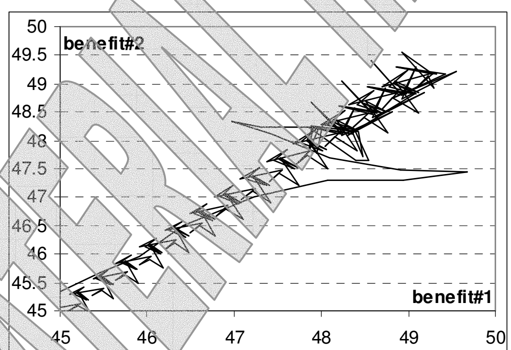
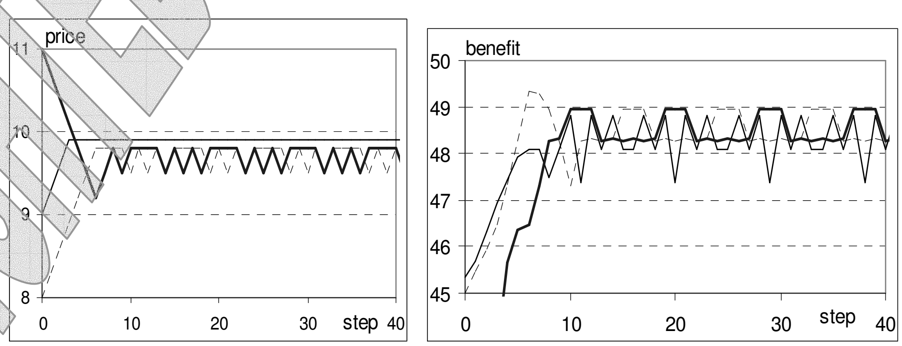


Fig. 26. Benefit co-evolution of the firm #1 and the firm #2 (case CCC, fuzzy increment)

The next three graphics obtained for a different matrix of delays [[0 3 4] [1 0 2][3 2 0]]. The initial prices of the companies are 8,9,11; the fixed increment 0.3, and the strategies CMC. In this case the system enters in a loop of period 9 after 11 steps (see Fig.27).



a) Price variation (case CMC-increment 0.3) b) Benefit variation (case CMC-increment 0.3)

Fig. 27. The evolution of the prices and the profits of three competitors

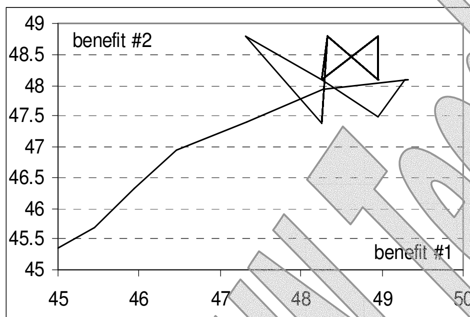
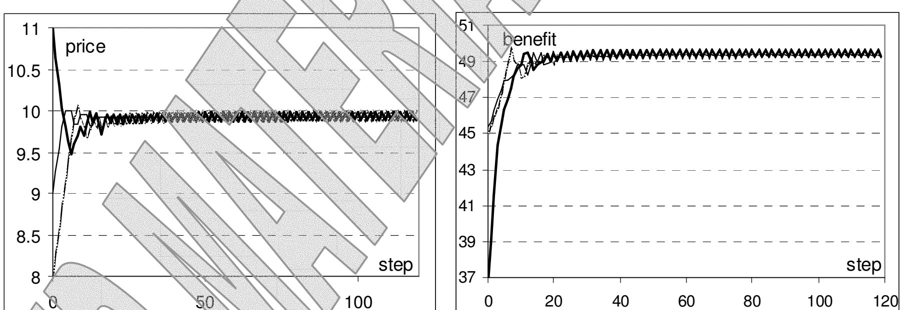


Fig. 28. Benefit co-evolution of the firm #1 and the firm #2

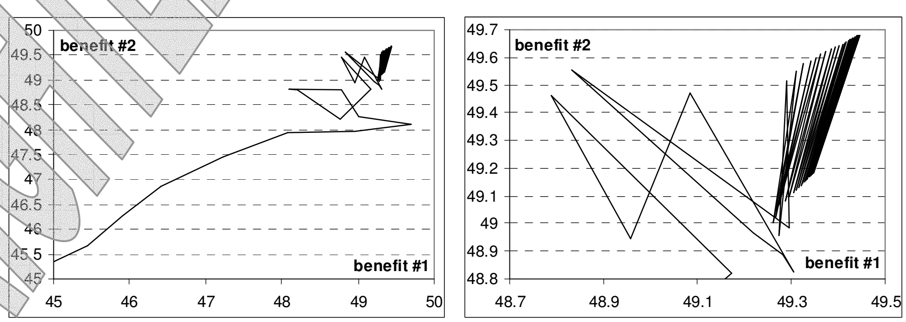
The corresponding co-evolution of the benefits is represented in Fig. 28.

For the same input data, changing the crisp increment into a fuzzy increment, the system enters in a regime looking like a dumped oscillation. The regime ends after 113 steps, when the behavior becomes periodic. The long transitory regime and the slowly modification of the prices and benefits can be noticed in the co-evolution graphs – see Fig. 30 and Fig. 31.



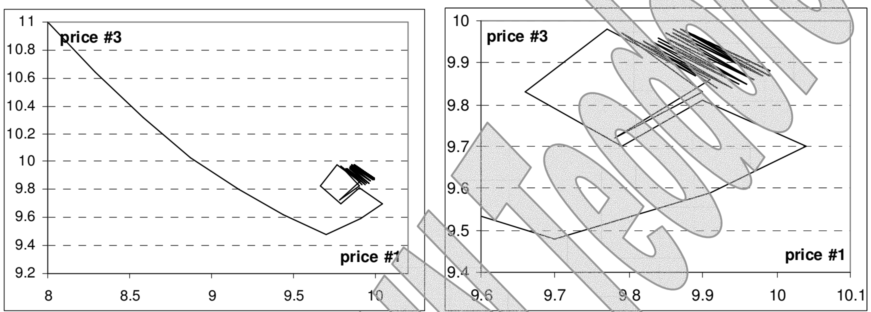
(a) (b)

Fig. 29. The evolution of the prices and the profits of three competitors (a) Price variation (case CMC, fuzzy increment); (b) Benefit variation (case CMC, fuzzy increment)



a) (b)

Fig. 30. a) Benefit co-evolution of the firm #1 and the firm #2; (b) details of the loop



(a) (b)

Fig. 31. (a) Price co-evolution of the firm #1 and the firm #3; (b) details of the loop

Changing the strategy (case CCC instead of CMC) and keeping unchanged all the other characteristics of the three vendor network (i.e., preserving the matrix of delays [[0 3 4][**1** 0 2][**3** 2 0]], the fuzzy increment type, and the initial prices 8,9,11) the system will enter in a loop of period 6 after 34 steps (20 steps if we omit the insignificant differences in the price and benefit variations.) The situation is graphed in Fig. 32, Fig. 33 and Fig. 34.

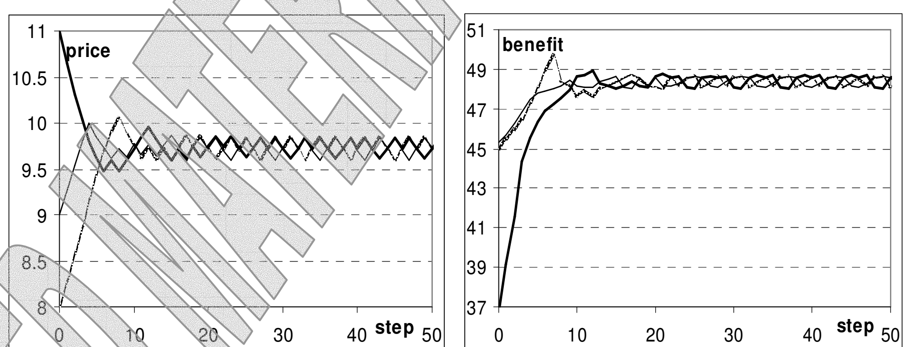


Fig. 32. The evolution of the prices and the profits of three competitors
(a) Price variation (case CCC, fuzzy increment) (b) Benefit variation (case CCC, fuzzy increment)

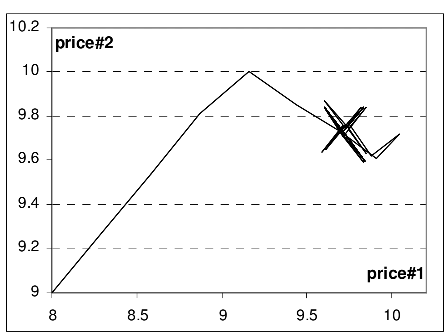
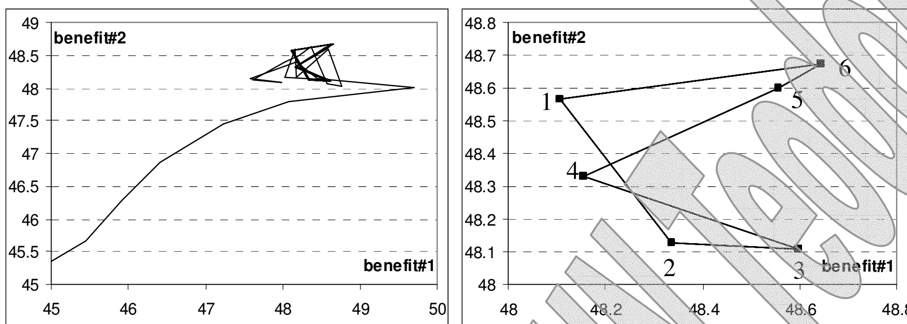


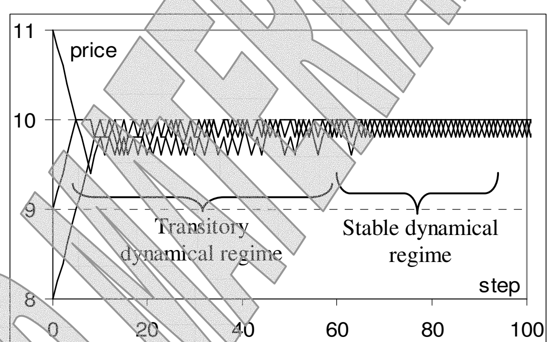
Fig. 33. Price co-evolution of the firm #1 and the firm #2



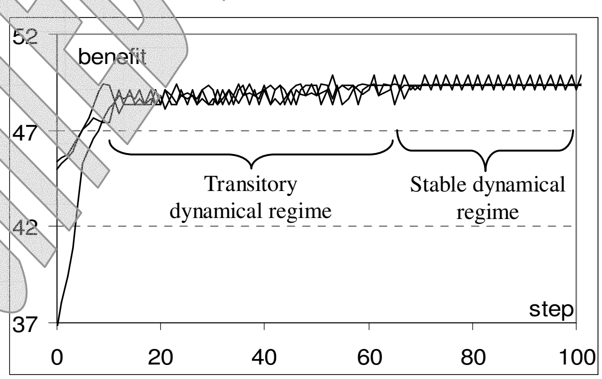
(a) (b)

Fig. 34. Benefit co-evolution of the firm #1 and the firm #2; (b) details of the loop

The graphics shown in Figs. 35, 36, 37, and 38 have resulted from simulations which have as input parameter the initial prices of the companies: 8, 9, 11; the strategies used in the simulation have been, for the three players, comp-benefit, max-benefit, and comp-benefit, respectively; the corresponding delays have been [[0 3 4][2 0 2][3 3 0]], and the fixed increment has been 0.2.



a) Price variation (case CMC, increment 0.2)



b) Profit variation (case CMC, increment 0.2)

Fig. 35. The evolution of the prices and the profits of three competitors

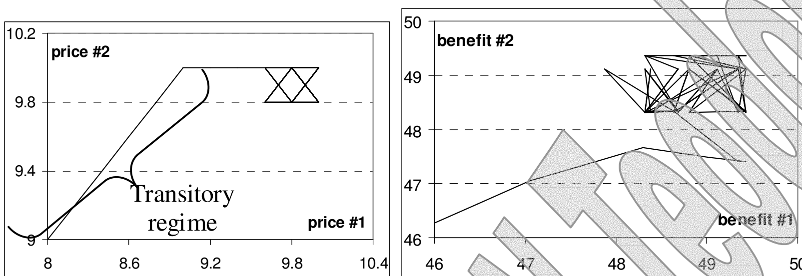


Fig. 36. (a) Price co-evolution of the firm #1 and firm #2; (b) Profit co-evolution of the firm #1 and firm #2

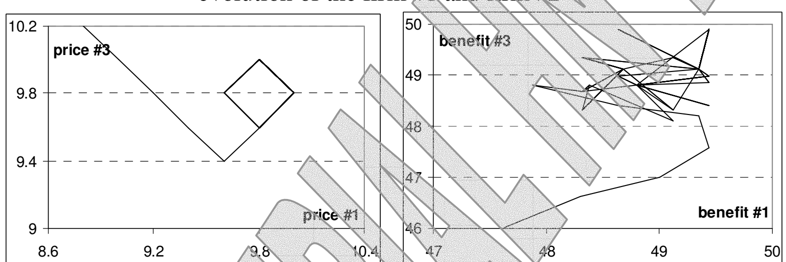


Fig. 37. (a) Price co-evolution of the firm #1 and firm #3; (b) Profit co-evolution of the firm #1 and firm #3

Fig. 38. (a) Price co-evolution of the firm #2 and firm #3. (b) Profit co-evolution of the firm #2 and firm #3

The presence on the market of a larger number of companies that sell the same product may result in a transitory process of smaller period, i.e., to a faster stabilization of the system, compared to the case of only two vendors present in the market. In simulations with several companies, no stabilization of the prices (and profits) after evolutions of 200 or even 250 steps was obtained. We recall that in the case of an economical micro-system with only two companies, stabilization has always been obtained.

For delays that may generate, in a two-system model, such an asymptotically stable behavior (stabilization number of steps $\tau = 3$, $\tau = 4$), for models including only three companies, we obtained stabilization for the most unfavorable case in about 70 steps. This is the situation represented in Fig. 37, where the price-benefit ratios of the firm #1 and firm #3 stabilizes in a loop of period-2 cycles. On the other hand, the benefit reaches a constant value in case of firm #2, the only one which uses the strategy *max-benefit*.

The behavior of this economic system is dynamical and nonlinear (see Fig. 35) and includes a transitory regime and an asymptotically dynamically stable regime. The graphs in Figs. 35-38 incorporate the co-evolution of the systems for 100 steps, a number of steps large enough for the transitory regime to vanish and the dynamically stable regime to be significantly present in the results (after 70 steps the system enters in the transitory regime). It is easy to visually detect the transitory regimes in the graph.

Table 3

Comparison of the behaviors of economic system with a number of 3 companies

Strategy of firms	Delays	Increment	System evolution
comp, max, comp (CMC)	{{0, 3, 4} {1, 0, 2} {3, 2, 0}}	fuzzy	Almost-loop of period 2 reached after 20 steps. Slight decrease in the oscillation amplitude for at least 113 steps (computed number of steps)
		0.1	Loop of period 2, after 27 steps
		0.2	Loop of period 2, after 24 steps
		0.3	Loops of period 9, after 11 steps
max, comp, max (MCM)		fuzzy	Loop of period 2, after 23 steps
		0.1	Stable, after 44 steps
		0.2	Stable, after 24 steps
		0.3	Loops of period 2, after 9 steps
comp, comp, comp (CCC)		fuzzy	Loop of period 6, after 34 steps
		0.1	Loop of period 2, after 26 steps
		0.2	Loop of period 2, after 23 steps
		0.3	Loop of period 2, after 13 steps
max, max, max (MMM)		fuzzy	Stable, after 15 steps
		0.1	Stable, after 28 steps
		0.2	Stable, after 18 steps
		0.3	Stable, after 9 steps
comp, max, comp (CMC)	{{0, 3, 4} {2, 0, 2} {3, 3, 0}}	fuzzy	Loop of period 2, after 85 steps
		0.1	Loop (2 1 2), after 27 steps
		0.2	Loop (2 1 2), after 70 steps
		0.3	Loop of period 9, after 7 steps
max, comp, max (MCM)		fuzzy	Loop 2, after 19 steps
		0.1	Stable, after 57 steps
		0.2	Stable, after 26 steps
		0.3	Loop of period 2, after 11 steps
comp, comp, comp (CCC)		fuzzy	Instable. After 685 the system automatically stops*
		0.1	Stable, after 595 steps
		0.2	Loop of period 12, after 14 steps
		0.3	Stable, after 220 steps
max, max, max (MMM)		fuzzy	Stable, after 13 steps
		0.1	Stable, after 29 steps
		0.2	Stable, after 20 steps
		0.3	Stable, after 10 steps

* The lower limit of the universe of discourse is reached (all prices are 1). The simulation automatically stops, signaling that the vendors are bankrupt.

Table 3 summarizes the behaviors of several systems with three companies, using different strategies, which have the initial prices 8, 9 and 11, and a fixed increment.

4. Comparison of 3-vendor case with N-vendor cases

Further results presented in [21] have been obtained on a generalized economic model. The model refers to a number of N companies, which may use strategies of various types. In this way, the implemented model is more general. The previous restriction, which requires that all the companies have to utilize the same type of strategy, was too restrictive in certain situation.

The evolution of the prices and profits, for a 5-vendor market, is briefly discussed in this section, and contrasted with the three vendor case.

Table 4

Behavior of 5-vendor networks

Initial prices	Strategies		Delays				
8	comp		0	4	4	4	4
8	comp		3	0	3	3	3
8	comp		2	3	0	2	3
8	comp		2	2	2	0	2
8	comp		1	1	1	1	0
Increment	0.3						

The firm #2 and the firm #5 have equal benefits. The firm #1 and firm #4 has a stable oscillate regime. The system enters in a loop of period 2 after 64 steps.

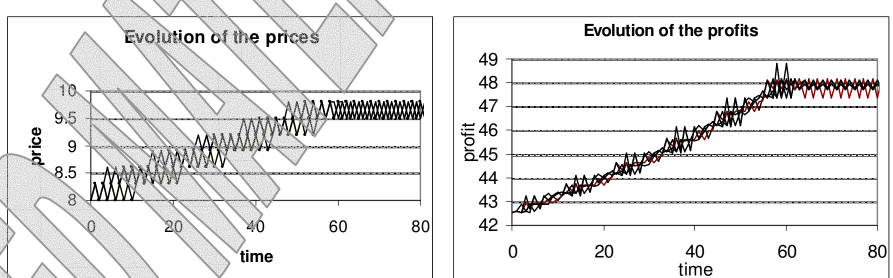


Fig. 39. The evolution of the prices and the profits in a network of five competitors

These simulations show that the number of vendors in the model does not play an important role.

5. Implementation issues

The implementation of the model under FuzzyCLIPS has several advantages and drawbacks, compared to an implementation under C/C++ or other usual high level programming languages.

While CLIPS means "C Language Integrated Production System", that is, it is based on C, the main difference in implementing under FuzzyCLIPS in contrast to C/C++ is that the first is a declarative language, like Prolog, while C/C++ is a procedural language. This means that the development work needed under CLIPS for describing "common" knowledge is lower, at least when implementing rules like "IF…THEN…" rules and their manipulation. However, FuzzyCLIPS is not well suited to perform analytic computations and to represent algebraic expressions (procedural tasks). Instead, CLIPS and its version FuzzyCLIPS have been developed to help represent various types of knowledge and especially to develop expert systems based on rules, making profit of the incorporated inference machine in CLIPS (see http://www.ghg.net/clips/CLIPS.html for a full introduction to language CLIPS). Therefore, to improve the models and the simulation capabilities, a transition to C or other procedural language is needed.

6. Conclusions and further research

In this paper, we have reported research on models of small systems of vendors whose strategies of price adjusting is either a rational one – profit maximization – or is highly subjective – aiming to obtain more gain then the competitors only. The second type of behavior, although driven by subjective judgment, is not uncommon in the market, especially when vendors are unfair and try to eliminate the competition. This behavior may be expected for uneducated or un-experienced players in the market, or players driven by impulsive, "envy-driven" aims.

The simulations show that the presence of a dominant number of companies that utilize the strategy *comp-benefit* (characterized by a "selfish" behavior) may lead to stabilization of the economic system in loops of larger periods (for example 9 steps). On the other side, the presence of a dominant number of companies that use the strategy *max-benefit* leads to smaller stabilization time (smaller transitory phase), moreover to a stabilization in a loop with smaller periods (usually, period-2 cycles), or even into a constant value. These dynamical tendencies are significant and show, at least in our simulations, and according to our models of behavior, that a rational behavior has beneficent role in the market dynamic stability. While this behavior has probably been intuited in the economic world for a long time, our results may be the first to provide a demonstration for a specific model.

This research and previous research demonstrate that a system with fuzzy increment, even in the case with only two companies, is always faster stabilized than a system with fixed increment. Another characteristic of this model of economic system is that the stable regime yields a constant value for a fuzzy increment, instead of a stable loop (oscillatory stable regime). This is true not only for systems comprising two vending companies, but for systems with more than two companies as well.

Further research should follow several paths. A detailed analysis of the overall model in a generalized case, with the fuzzy logic definition of the price and profit using adjustable membership functions, is an obvious way to further the analysis. Addition of constraints, for example to the price variation is also needed. Indeed, the price in the simulation presented here can decrease up to zero, which is unrealistic, because the vendors are assumed to pay a minimal amount for the sold products and thus the selling prices can not be lower than that minimal amount. Further refinements of the strategies are also needed, to improve the modeling of the market. At this level, the modeling is in its early stages, but we believe the modeling results are useful because they obviate dynamical processes that have not been explained until now, at our best knowledge. Also, the modeling method and the approach itself are new.

Acknowledgements

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